Variance Estimation
Lemma 4.1:

Suppose $z$ is a random vector with

$$E(z) = \mu \quad \text{and} \quad \text{Var}(z) = \Sigma.$$  

Then $E(z'Az) = \mu' A \mu + tr(A \Sigma)$. 
Result 4.2:

Under the GMM, an unbiased estimator of $\sigma^2$ is

$$\hat{\sigma}^2 = \frac{SSE}{n - r},$$

where $r = \text{rank}(X)$ and

$$\text{SSE} = \hat{\varepsilon}'\hat{\varepsilon} = (y - \hat{y})'(y - \hat{y}) = [(I - P_X)y]'(I - P_X)y$$

$$= y'(I - P_X)'(I - P_X)y = y'(I - P_X)(I - P_X)y$$

$$= y'(I - P_X)y$$

$$= \text{“Sum of Squared Errors.”}$$
The “Regression Sum of Squares” or “Sum of Squares for Regression” is

\[ SSR = \hat{y}'\hat{y} = (P_{XY})'P_{XY} \]
\[ = y'P'_{X}P_{XY} \]
\[ = y'P_{XY}. \]

Find \( E(\text{SSR}) \).
Show that the Total Sum of Squares $y'y = SSR + SSE$. 