Variance Estimation

Lemma 4.1:

Suppose z is a random vector with

$$E(z) = \boldsymbol{\mu} \quad \text{and} \quad \mathrm{Var}(z) = \boldsymbol{\Sigma}.$$

Then $\boldsymbol{E}(z'\boldsymbol{A}z) = \boldsymbol{\mu}'\boldsymbol{A}\boldsymbol{\mu} + tr(\boldsymbol{A}\boldsymbol{\Sigma})$.

Result 4.2:

Under the GMM, an unbiased estimator of σ^2 is

$$\hat{\sigma}^2 = \frac{\text{SSE}}{n - r},$$

where $r = rank(\boldsymbol{X})$ and

$$\begin{aligned}
\text{SSE} &= \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}} = (\boldsymbol{y} - \hat{\boldsymbol{y}})'(\boldsymbol{y} - \hat{\boldsymbol{y}}) = [(\boldsymbol{I} - \boldsymbol{P_X})\boldsymbol{y}]'(\boldsymbol{I} - \boldsymbol{P_X})\boldsymbol{y} \\
&= \boldsymbol{y}'(\boldsymbol{I} - \boldsymbol{P_X})'(\boldsymbol{I} - \boldsymbol{P_X})\boldsymbol{y} = \boldsymbol{y}'(\boldsymbol{I} - \boldsymbol{P_X})(\boldsymbol{I} - \boldsymbol{P_X})\boldsymbol{y} \\
&= \boldsymbol{y}'(\boldsymbol{I} - \boldsymbol{P_X})\boldsymbol{y} \\
&= \text{“Sum of Squared Errors.”}
\end{aligned}$$

The “Regression Sum of Squares” or “Sum of Squares for Regression” is

$$\begin{aligned} \text{SSR} = \hat{\boldsymbol{y}}'\hat{\boldsymbol{y}} &= (\boldsymbol{P}_X\boldsymbol{y})'\boldsymbol{P}_X\boldsymbol{y} \\ &= \boldsymbol{y}'\boldsymbol{P}_X'\boldsymbol{P}_X\boldsymbol{y} \\ &= \boldsymbol{y}'\boldsymbol{P}_X\boldsymbol{y}. \end{aligned}$$

Find $E(\text{SSR})$.

Show that the Total Sum of Squares $\boldsymbol{y}'\boldsymbol{y} = \mathrm{SSR} + \mathrm{SSE}$.