1. Consider an experiment designed to determine which of four soup recipes consumers prefer most. A total of 24 volunteer tasters were used in the experiment. Each volunteer tasted three of the four soup recipes in an assigned order and provided a score for each recipe using a 9-point scale (1 = poor, ..., 9 = excellent). Three batches of soup were prepared using each of the four recipes. Each of the 12 total batches of soup was divided into six bowls. These 72 total bowls were tasted by the 24 tasters (three bowls per taster). Data are available at [http://www.public.iastate.edu/~dnett/S511/soup.txt](http://www.public.iastate.edu/~dnett/S511/soup.txt).

Use `lmer` to fit a mixed effects linear model to these data. Include recipe and taste order as fixed factors. Include taster and batch as a random factors. Use R to help you complete the following parts.

(a) Provide REML estimates of the model variance components.

(b) Estimate the correlation between the first and seventh observations in the data set based on the model fit.

(c) Determine the BLUP of the random effect associated with taster 1. You may wish to use the command `ranef(o)` to see BLUPs from `lmer` output stored in the object `o`.

(d) Determine the lsmean for recipe 1. You may wish to use the command `fixef(o)` to see parameter estimates from `lmer` output stored in the object `o`.

(e) Provide the standard error for the estimate in part (d). You might wish to use `vcov(o)`.

(f) Provide an estimate of the difference between means for recipes 1 and 2.

(g) Provide the standard error for the estimate in part (f).

(h) Based on the AIC criterion, should the model include interaction between the factors recipe and taste order? Give the AIC values you used to answer this question.

(i) Conduct a likelihood ratio test to answer the question in part (h). State the test statistic, the degrees of freedom, the $p$-value, and a conclusion.

2. Suppose independent and identically distributed draws from a $N(\mu, \sigma^2)$ distribution are observed to be 31.4, 30.6, 28.6, 27.5, 29.6. Write R code that will allow you to plot the profile log likelihood function for $\mu$ over values for $\mu$ ranging from 24 to 35. Turn in your code and your plot of the profile log likelihood function. Also, report the value of the profile log likelihood at $\mu = 28$.


(a) Examine slide 34. There is a column labeled `df` in the output from the `anova` command. Explain why the numbers reported there are 49 and 23?

(b) Use the object `d.timemle` to estimate the coefficients of the quadratic equation relating mean strength to time for each of the three programs.

(c) Use the object `d.timer` to obtain predictions of the quadratic equation relating mean strength to time for the first subject in the dataset and for the last subject in the dataset.

4. An experiment was designed to compare the effect of three drugs (A, B, and C) on the heart rate of women. Five women were randomly assigned to each drug. The heart rate (in beats per minute) of each woman was measured at 0, 5, 10, and 15 minutes after the drug was administered. The data are provided in the file
(a) Which of the three correlation structures – compound symmetry, AR(1), or unstructured – seem most appropriate for these data? Support your answer with appropriate analyses.

(b) Using the correlation structure selected in part (a), provide an analysis of the data to help researchers understand the relationship between heart rate and the factors drug and time. Produce plots, tests, and/or confidence intervals as necessary to support your conclusions.

5. In an enzyme kinetics study, researchers expected the mean velocity of a reaction to be related to concentration \((x)\) through the relationship

\[
\text{mean velocity} = \frac{\beta_1 x}{(\beta_2 + x)},
\]

where \(\beta_1\) and \(\beta_2\) are unknown parameters.

(a) Use the data set

http://www.public.iastate.edu/~dnett/S511/EnzymeKinetics.txt

to estimate \(\beta_1\) and \(\beta_2\).

(b) Estimate the concentration at which the mean reaction velocity is equal to 15.

(c) Provide an approximate 95% confidence interval to accompany your point estimate in part (b).