

**STAT 511 Homework 8**  
**Due Date:** 11:00 A.M., Friday, March 30

1. A meat scientist is studying the effect of storage temperature on meat quality. The temperatures of interest are 34, 40, and 46 degrees Fahrenheit. Twelve coolers are available for the study. The three temperatures are randomly assigned to the twelve coolers using a balanced and completely randomized design. Two large cuts of fresh beef are stored in each cooler. After three days, each member of a team of experts independently assigns a quality score to each cut of beef. The experts are not told about the storage conditions of each cut. The scores assigned by the team to each cut of beef are averaged to produce an overall quality score for each cut.
  - (a) Write down a model for the overall quality score data. Define your notation thoroughly.
  - (b) Write down an ANOVA table for the overall quality score data. Include Source, Degrees of Freedom, and Sums of Squares columns.
  - (c) Describe one test you would conduct with the overall quality score data. Indicate how you would compute the test statistic and state its degrees of freedom.
2. Recall the split-plot experimental design described on slides 30 to 39 of the slide set entitled *Correspondence between Experimental Designs and Mixed-Effect Models*. Review the answers to Homework 7 Problem 2, and then run the code provided at

<http://www.public.iastate.edu/~dnett/S511/HW8SimulateSplitPlot.R>

to simulate data from such an experiment. Examine the code and conduct analyses of the simulated data to answer the following questions.

- (a) What are the true means for each combination of diet and drug according to the simulation code?
- (b) What are the true values of the variance components according to the simulation code?
- (c) Use the functions `lme` and `anova` to produce an ANOVA table for the simulated data. Use the values from this ANOVA table to help you complete the following parts.
  - i. Use observed mean squares to compute the values of unbiased estimators of the variance components.
  - ii. Provide an approximate 95% confidence interval the difference between the diet 1 drug 1 mean and the diet 2 drug 1 mean (an interval for comparing diet 1 to diet 2 among mice receiving drug 1).
  - iii. Conduct an  $F$  test for diet main effects. Compute the test statistic, state its degrees of freedom, and determine a  $p$ -value.
- (d) Determine maximum likelihood estimates of the variance components.
- (e) Determine REML estimates of the variance components.
- (f) Use `lme` and the `anova` function conduct an  $F$  test for diet main effects. Compute the test statistic, state its degrees of freedom, and determine a  $p$ -value.

3. Suppose

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_1 \\ \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \sigma^2/2 & 0 \\ \sigma^2/2 & \sigma^2 & \sigma^2/2 \\ 0 & \sigma^2/2 & \sigma^2 \end{bmatrix} \right)$$

where  $\mu_1 \in \mathbb{R}$ ,  $\mu_2 \in \mathbb{R}$ , and  $\sigma^2 > 0$  are unknown parameters. Find the REML estimator of  $\sigma^2$ .

4. Consider an experiment designed to determine which of four soup recipes consumers prefer most. A total of 24 volunteer tasters were used in the experiment. Each volunteer tasted three of the four soup recipes in an assigned order and provided a score for each recipe using a 9-point scale (1 = poor, . . . , 9 = excellent). Three batches of soup were prepared using each of the four recipes. Each of the 12 total batches of soup was divided into six bowls. These 72 total bowls were tasted by the 24 tasters (three bowls per taster). Data are available at <http://www.public.iastate.edu/~dnett/S511/soup.txt>.

Use `lmer` to fit a mixed effects linear model to these data. Include recipe and taste order as fixed factors. Include taster and batch as a random factors. Use R to help you complete the following parts.

- (a) Provide REML estimates of the model variance components.
- (b) Estimate the correlation between the first and seventh observations in the data set based on the model fit.
- (c) Provide the BLUP of the random effect associated with taster 1.
- (d) Provide an estimate of the mean score for recipe 1.
- (e) Provide the standard error for the estimate in part (d).
- (f) Provide an estimate of the difference between means for recipes 1 and 2.
- (g) Provide the standard error for the estimate in part (f).
- (h) Based on the AIC criterion, should the model include interaction between the factors recipe and taste order? Give the AIC values you used to answer this question.
- (i) Conduct a likelihood ratio test to answer the question in part (h). State the test statistic, the degrees of freedom, the  $p$ -value, and a conclusion.