STAT 511 FINAL EXAM SOLUTIONS

SPRING 2010

1a) $1+6+2.5 = 9.5$

b) $t = 0.602$ $d.f. = 60-4 = 56$

p-value $\cong 0.55$

The mean response for Treatment 1 was not significantly different from 0.

c)

EFFECT DIFF	ESTIMATE	SE	t-stat	$P \le 0.05$
1 - 2	-4	2.036	-1.965	No
1 - 3	-6	2.036	-2.948	YES
2 - 3	-2	2.036	$-2/2.036$	NO

d) $\widehat{drug2} = \bar{Y}_{.2.} - \bar{Y}_{.1.}$

$\hat{Var}(\widehat{drug2}) = \hat{\sigma}^2/30 + \hat{\sigma}^2/30 = \hat{\sigma}^2/15$

1 d) (continued)

$$SE(\hat{drug2}) = \hat{\sigma}/\sqrt{15}$$

From the R output,

$$SE(\hat{drug2}) = 1.662.$$

Thus, $\hat{\sigma}^2 = (1.662\sqrt{15})^2$

$$= 1.662^2 * 15.$$

1 e) d.f. = 60 − 4 = 56

2 a) 1 0 0 0 0 0

1 0 1 1 0 1

b) $1\ 0\ 0\ 0\ 0\ 0\ 1\ \underbrace{0\ 0\ 0\ 0 \cdots 0}_{11 \text{ zeros}}$

$0\ 0\ 0\ 0\ 0\ 1\ \overbrace{0\ 0\ 0 \cdots\cdots 0}\ 1$

c) $\left[\underset{6\times6}{I} \otimes \underset{4\times1}{\underset{\sim}{1}},\ \underset{12\times12}{I} \otimes \underset{2\times1}{\underset{\sim}{1}}\right]$

d) σ_s^2 = Variance of subject random effects

σ_r^2 = Variance of ear random effects

σ_e^2 = Error Variance.

e) $\sigma_s^2 + \sigma_r^2 + \sigma_e^2$

f) $\dfrac{\sigma_s^2 + \sigma_r^2}{\sigma_s^2 + \sigma_r^2 + \sigma_e^2}$

2 g) $$\frac{\sigma_s^2}{\sigma_s^2 + \sigma_r^2 + \sigma_e^2}$$

2 h) There is a mean for each combination of hearing aid and test. We should expect the pre-test means to be identical across hearing aid types because the pretest data is collected before the hearing aids are placed. Thus, rather than 6 means, we should have only 4 (pretest and postest for each of the 3 hearing aid types).

3. a) Model 2

b) $128.10 - 2(11) + \log(20)(11)$

c) i. $42.834 - 14.397$

ii. $11 - 1 = 10$

iii. Model 2 does fit significantly better. The test stat is far bigger than $18.307 = \chi^2_{10}(.95)$.

d) i. $38.69 - 14.397$

ii. $11 - 2 = 9$

iii. Again Model 2 fits significantly better.

3 e) The mean number of infected cells for plants of Genotype 2 was estimated to be $\exp\{0.22450\}$ times the mean number of infected cells for plants of genotype 1.

3 f) $\exp\{2.68785 + 0.22450\}$

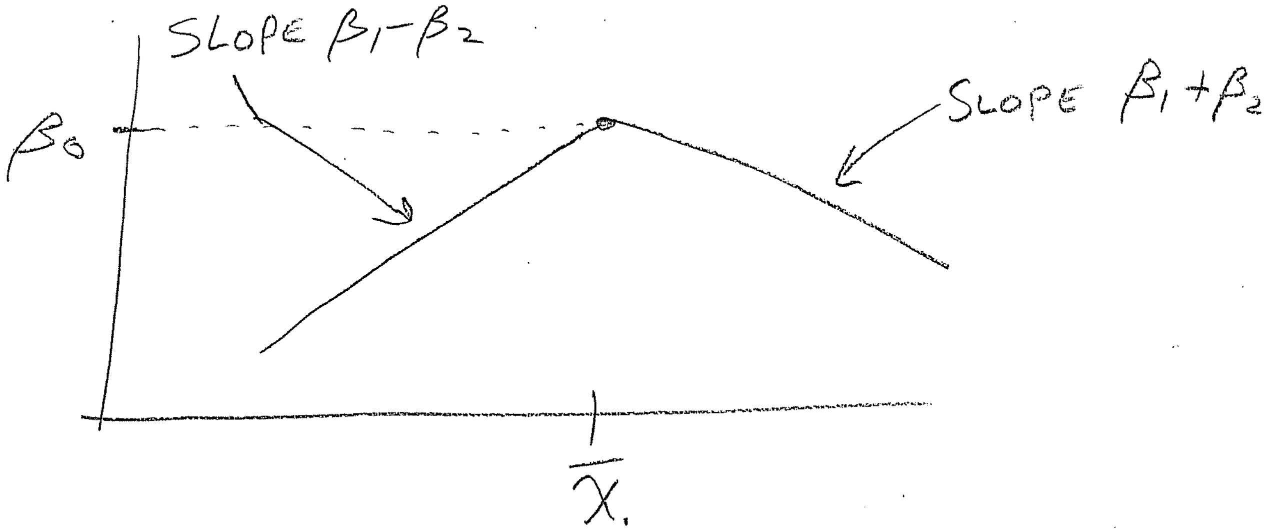
4. $(X'X + \lambda^2 D)^{-1} X' \underset{\sim}{y}$ where $D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$X'X = \begin{bmatrix} n & 0 \\ 0 & \sum_{i=1}^{n} (x_i - \bar{x}_.)^2 \end{bmatrix} \qquad X'\underset{\sim}{y} = \begin{matrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} (x_i - \bar{x}_.) y_i \end{matrix}$$

$$\hat{\underset{\sim}{\beta}}_{\lambda^2} = \begin{bmatrix} \bar{y}_. \\ \dfrac{\sum_{i=1}^{n} (x_i - \bar{x}_.) y_i}{\sum_{i=1}^{n} (x_i - \bar{x}_.)^2 + \lambda^2} \end{bmatrix}$$

5. $\beta_0 + \beta_1 (x_i - \bar{x}_.) + \beta_2 |x_i - \bar{x}_.|$

$$= \begin{cases} \beta_0 + (\beta_1 - \beta_2)(x_i - \bar{x}_.) & \text{if } x_i \le \bar{x}_. \\ \beta_0 + (\beta_1 + \beta_2)(x_i - \bar{x}_.) & \text{if } x_i > \bar{x}_. \end{cases}$$



This is a continuous, piecewise linear spline function with a knot at $\bar{x}_.$.

6 a) 0.1142

b) 0.0128

c) 0.1113 – 0.1142

d) $0.1142-(0.1113-0.1142)$

$= 2(0.1142) - 0.1113$

e) 0.0898 To 0.1323