Analysis of a Split-Plot Experiment with Whole-Plot Part as RCBD

An experiment was conducted to measure the effect of two factors,

$$A = \text{row spacing} \qquad \text{and} \qquad B = \text{plant density},$$

on a complex response variable y. The response variable is related to the relationship between a plant's leaf area and the amount of light intercepted at various levels in its canopy. Destruction of several plants is necessary to obtain a single value of y.

A field was divided into $r = 4$ blocks. Each block was divided into two *whole plots*, and $a = 2$ row spacings (38 and 76 cm) were randomly assigned to the whole plots within each block. Each whole plot was divided into four *split plots*, and $b = 4$ plant densities were randomly assigned to the split plots within each whole plot. At a predetermined date during the growing season, the split plots were used to obtain $rab = (4)(2)(4) = 32$ measures of the response variable. We can consider the following model for the data

$$\begin{aligned} y_{ijk} &= \mu + \rho_k + \alpha_i + (wp)_{ik} && \text{(whole-plot portion)} \\ &\quad +\beta_j + (\alpha\beta)_{ij} + (sp)_{ijk} && \text{(split-plot portion)} \\ &\quad (i = 1, \ldots, a \quad j = 1, \ldots, b \quad k = 1, \ldots, r) \end{aligned}$$

where $(wp)_{ik} \sim N(0, \sigma^2_{wp})$, $(sp)_{ijk} \sim N(0, \sigma^2_{sp})$, and all random effects are independent. We may partition as follows:

Whole Plot Partitioning

SOURCE	DF	DF
Block	$r-1$	3
A	$a-1$	1
W.P. Error	$(r-1)(a-1)$	3
C. Total(wp)	$ra-1$	7

Split Plot Partitioning

SOURCE	DF	DF
Whole Plot	$ra-1$	7
B	$b-1$	3
AB	$(a-1)(b-1)$	3
S.P. Error	$(r-1)a(b-1)$	18
C. Total(sp)	$rab-1$	31

W.P. Error is Block$*A$.

S.P. Error is the usual error term, which happens to consist of Block$*B$+Block$*A*B$.

SAS code for the analysis using *proc glm* and *proc mixed* is provided below (and in *splitplot.sas*). Output can be found on the back of this sheet.

```
proc glm;
  class block spacing density;
  model y=block spacing block*spacing
          density spacing*density;
  random block*spacing;
  test h=spacing e=block*spacing;
run;

proc mixed method=type3;
  class block spacing density;
  model y=block spacing
          density spacing*density / ddfm=satterthwaite;
  random block*spacing;
  lsmeans spacing density;
  lsmeans spacing*density / slice=spacing;
  lsmeans spacing*density / slice=density;
run;
```