# Some Notes on Mixed Models

A *mixed-effects model* or simply a *mixed model* is a model that includes a mixture of fixed and random factors. Recall that each factor in an experiment has levels. The effects associated with a factor are the effects that the levels of the factor have on the response variable of interest.

## Fixed vs. Random

Generally speaking a factor is *fixed* if the levels of the factor were selected by the investigator with the purpose of comparing the effects of the levels to one another. One of the major goals of the analysis is to test for differences among the effects associated with the specifically chosen levels of the factor and to describe the specific differences that exist.

A factor is *random* if the effects associated with the levels of the factor can be viewed as being like a random sample from a population of effects. For random effects, we can make statements about variation in the population of random effects from which the effects at hand are considered to be like a random sample. Furthermore, we can generalize our conclusions about fixed factors to the populations associated with random factors. We are usually not interested in comparisons among the levels of random effects. Rather, we are interested in studying variation in the population from which the random effects are like a random sample or in controlling for that variation so that proper conclusions about fixed effects can be drawn.

An interaction between or among factors is considered to be random if any one of the factors involved in the interaction is random.

## Crossed vs. Nested

Two factors *A* and *B* are said to be *crossed* if the levels of *B* are the same for all levels of *A*.

Factor *B* is said to be nested within factor *A* if the levels of *B* are not identical for all levels of *A*.

## Specifying Terms in Mixed Models for the Analysis of Split-Plot Experiments

Main effects for fixed factors and all possible interactions among fixed factors are typically included. A random term is included if the term represents either a blocking factor, experimental units, or observational units. Random interaction terms that do not correspond to experimental units or observational units are not included unless the interaction is of interest or expected to be large. According to the mixed model, observations that share a level of a random effect are positively correlated. The more random effect levels shared by two observations, the greater the positive correlation among those observations. It is sometimes useful to think about the expected level of correlation between observations when deciding whether to include a random interaction term in the model. Finally, it is not possible to consider an interaction between a factor and another term that involves nesting within that factor. For example, C*B(A) might be included, but A*B(A) could not be considered.

## Error Terms in Mixed Models for the Analysis of Split-Plot Experiments

The error term for testing for differences among the levels of a fixed factor will typically be the random term associated with the experimental units to which the levels of the factor were randomly assigned. The error term for testing for significant interaction will usually be the error term with the most degrees of freedom among the error terms for the factors in the interaction.

An experiment was conducted to compare the characteristics of bread made from two different recipes using two different types of flour and baked at three different temperatures. Three different people, each with their own oven, participated in the experiment. Each person independently prepared (in a random order) one batch of dough for each combination of recipe and flour type. Three loaves of bread were made from each batch. The three loaves from each batch were randomly assigned to the three baking temperatures and baked independently in a random order at their assigned temperatures. After baking, each loaf was cut into 10 slices. Response variables of interest were measured for each slice.

1. Provide an ANOVA table with columns labeled SOURCE and DF.

2. Circle the terms that are random.

3. Draw an arrow from each fixed term to the appropriate error term for testing the significance of the fixed term.

4. Name the crossed factors in this experiment.

5. How many random effect levels are shared by the observations from two slices of bread that come from

a) the same loaf

b) two loaves of bread baked at the same temperature by one person

c) two loaves of bread from the same batch

d) two loaves of bread made with the same type of flour using the same recipe and baked at the same temperature by two different people.