

1. An experiment was conducted to study the effects of 2 fertilizers ($F1$ and $F2$) on the yield of two corn varieties ($V1$ and $V2$). The portion of a field used for the experiment was divided into 24 plots. The 4 possible combinations of fertilizer and variety were assigned to the 24 plots in a balanced and completely randomized fashion. The average and variance of the 6 yields (in bushels) associated with each combination of fertilizer and variety are provided in the table below.

Fertilizer	Variety	Number of Plots	Average of Yields	Variance of Yields
$F1$	$V1$	6	42	8
$F1$	$V2$	6	40	6
$F2$	$V1$	6	36	5
$F2$	$V2$	6	39	9

- (a) A combination of fertilizer and variety is a treatment in this experiment. Is there any evidence to suggest that the treatment means differ in this case? Conduct an appropriate test to answer this question.

i. Test Statistic:

ii. Appropriate Table Value:

iii. Conclusion:

- (b) Is there significant interaction between fertilizer and variety? Conduct an appropriate test to answer this question.

i. Test Statistic:

ii. Appropriate Table Value:

iii. Conclusion:

- (c) The researchers are primarily interested in the effect of the fertilizers on yield. Construct an appropriate confidence interval or appropriate confidence intervals that will best help the researchers understand any significant differences that might exist between the fertilizers $F1$ and $F2$. Provide an explanation of your work that the researchers will be able to understand.

2. An experiment was conducted to compare the accuracy of two mass spectrometers (*A* and *B*) in measuring the ratio of ^{14}N to ^{15}N in soil. Eight plots were randomly selected from a large field containing many plots. Three soil samples were randomly selected from each plot. The ratio of ^{14}N to ^{15}N was measured twice for each sample, once using spectrometer *A* and once using spectrometer *B*. Code and output for the analysis of the data are provided below.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	31	199.9689462	6.45	5.14	0.0006
Error	16	20.0986270	1.26		
Corrected Total	47	220.0675732			

Source	DF	Type III SS	Mean Square	F Value	Pr > F
spect	1	5.69146948	5.69	4.53	0.0492
plot	7	88.82747507	12.69	10.10	<.0001
sample(plot)	16	24.85073864	1.55	1.24	0.3382
spect*plot	7	80.59926300	11.51	9.17	0.0001

Source	Type III Expected Mean Square
spect	$\text{Var}(\text{Error}) + 3\text{Var}(\text{spect}*\text{plot}) + \text{Q}(\text{spect})$
plot	$\text{Var}(\text{Error}) + 3\text{Var}(\text{spect}*\text{plot}) + 2\text{Var}(\text{sample}(\text{plot})) + 6\text{Var}(\text{plot})$
sample(plot)	$\text{Var}(\text{Error}) + 2\text{Var}(\text{sample}(\text{plot}))$
spect*plot	$\text{Var}(\text{Error}) + 3\text{Var}(\text{spect}*\text{plot})$

- (a) Is there a significant difference between the mean reading provided by spectrometer *A* and the mean reading provided by spectrometer *B*? Give an appropriate test statistic, a table value or p-value, and a conclusion.

- (b) Estimate the variance component associated with plot-to-plot variation in measurements.

- (c) Estimate the variance component associated with sample-to-sample variation in measurements made on samples within a plot.

- (d) What is another name for the source of variability labeled “error” in the output on the previous page?
3. Researchers are interested in studying the effects of a feed additive and an injected drug on the lean percentage of hogs. Twelve pens of hogs containing 2 hogs each are available for the experiment. Six of the 12 pens are randomly selected to receive the experimental feed additive throughout the experiment. The other 6 pens receive standard feed without the additive throughout the experiment. (Note that for practical reasons, the 2 hogs within any given pen must receive the same feed.) Within each pen, one hog is randomly selected to receive an injection containing the experimental drug while the other hog receives an injection of a saline solution that acts as a control.
- (a) Provide the SOURCE and DF columns of an ANOVA table for the analysis of this experiment.

- (b) Add a third column to the ANOVA table you started in part (a). Write “random” in each row corresponding to a random term. In the row for each fixed term, write the name of the error term that should be used to test the significance of the fixed term.
- (c) Now suppose that the 12 pens available for the experiment are spread over 6 farms with 2 pens on each farm. Suppose the 6 farms have been randomly selected from a large population of farms. Explain briefly how you would assign the feed additive treatment and drug treatment under this new scenario.

(d) For the new design you described in part (c), write down a table like the one created in parts (a) and (b).

4. Researchers at Iowa State are interested in measuring the effects of cold growing temperatures on two inbred corn lines (Mo17 and Co255). One measure of damage to corn plants is electrical conductivity. This response variable is obtained by measuring electrical conductivity through a deionized liquid that contains leaf tissue from a corn plant. A high electrical conductivity reading indicates that the leaf is leaking fluids into the deionized liquid. Thus high electrical conductivity readings are associated with high damage while low readings indicate low damage.

The researchers are interested in studying the effect of 3 temperatures (8°, 12°, and 24° C) and 2 external chemical treatments (U and V) on electrical conductivity in both Mo17 and Co255 plants. The researchers have 3 temperature-controlled growth chambers to work with. Each chamber can be set at any temperature. The chambers, however, are not identical, and there is some concern that some chambers are better at maintaining a constant temperature than others. The researchers grew 18 plants from the Mo17 line and 18 plants from the Co255 line. The 36 plants are contained in 36 separate pots. The researchers conducted one run of an experiment as follows.

The temperatures 8°, 12°, and 24° C were randomly assigned to the three chambers as depicted in the table below.

	Chamber A	Chamber B	Chamber C
Run 1	12	24	8
Run 2			
Run 3			

Six of the 18 Mo17 plants and 6 of the 18 Co255 plants were randomly selected for use in run 1. The other 24 plants were set aside for use in runs 2 and 3 of the experiment. The 6 selected Mo17 plants were randomly assigned to three chambers so that each chamber contained 2 Mo17 plants. The same process was repeated with the Co255 plants, so

that each chamber contained 2 randomly selected Co255 plants in addition to the 2 randomly selected Mo17 plants. In each chamber, one of the Mo17 plants and one of the Co255 plants were randomly chosen to be independently treated with chemical U . The other plants in the chambers were independently treated with chemical V . After leaving the plants in the temperature controlled chambers for 48 hours, the electrical conductivity of each of the plants was separately measured.

- (a) For runs 2 and 3 of the experiment, the researchers intend to assign plants to chambers and chemical treatments to plants using the same strategy employed in run 1. They are not sure, however, about the best way to assign temperatures to chambers in the second and third runs. Write assigned temperatures into the table on the previous page using an appropriate design strategy. Explain briefly the reasoning behind your design choice, and clearly explain how you used the table of random digits to assign temperatures to chambers in runs 2 and 3.

- (b) Provide the SOURCE and DF columns of the proper ANOVA table for the analysis of this experiment.

(c) The chambers used in this experiment are each large enough to hold 24 plants at one time. Thus the experiment could have been conducted in one single run. It seems like the data would be easier to analyze had there been only one run of the experiment. Briefly explain the most important drawback to the strategy of conducting the experiment using only a single run.