Introduction to Two-Factor Analysis of Variance

An experiment was conducted to study the effect of floor space and feeder trough space on pig growth. A total of 40 pens of pigs were randomly assigned to four treatments in a completely randomized fashion with 10 pens per treatment. The four treatments considered were

1) low floor space (0.3 m^2/pig) and low feeder trough space (2 cm/pig)
2) low floor space (0.3 m^2/pig) and high feeder trough space (4 cm/pig)
3) high floor space (0.6 m^2/pig) and low feeder trough space (2 cm/pig)
4) high floor space (0.6 m^2/pig) and high feeder trough space (4 cm/pig).

The treatments were imposed for 13 weeks beginning when pigs reached 17 days of age. The average weight gained (in kg) by the pigs in each pen was used as the response variable.

1. A **factor** is another name for an explanatory variable studied in an investigation. Name the two factors in this study.

2. The different values of a factor are called **levels**. What are the levels of each factor used in this experiment?

3. In a **full-factorial experiment**, the treatments consist of all possible combinations of levels from the factors being studied. Is this a full-factorial experiment?

4. The treatments are assigned to **experimental units**. What are the experimental units in this experiment?

5. The usual one-way ANOVA model was fit to the data from the experiment. The residual sum of squares was 555.6. The treatment means (in kg) were as follows.

		FEEDER TROUGH SPACE	
		low	high
FLOOR	low	56.4	55.8
SPACE	high	58.8	61.0

Is there any evidence that the treatments affected weight gain? Conduct one test to answer this question.

6. A general analysis strategy for a two-factor experiment involves first checking model assumptions by examining residuals as in single-factor ANOVA or multiple regression. After transforming the data (if necessary), analysis proceeds with a test for interaction between the two factors. There is **interaction** between two factors if the effect that one factor has on the response is not the same for all levels of the other factor. Is the effect of the floor-space factor the same for both levels of the feeder-trough-space factor? Conduct one test to answer this question.

7. When there is no significant interaction between factors, it makes sense to average over the levels of the second factor when making comparisons among the levels of the first factor. Conduct one test that compares the average of the low-floor-space treatments to the average of the high-floor-space treatments.

8. Likewise, when there is no significant interaction between factors, it makes sense to average over the levels of the first factor when making comparisons among the levels of the second factor. Conduct one test that compares the average of the low-feeder-trough-space treatments to the average of the high-feeder trough-space treatments.

9. Suppose the high-floor-space and low-feeder-trough-space mean had been 50.8 kg instead of 58.8 kg. Conduct the test for interaction again using this new data.

10. Explain why the tests conducted in 7 and 8 might no longer be of interest. What tests might be considered instead?

```
proc glm;
  class floor feeder;
  model y=floor feeder floor*feeder / clparm;
  lsmeans floor feeder floor*feeder;
  estimate 'H floor - L floor' floor 1 -1;
  estimate 'H feeder - L feeder' feeder 1 -1;
run;
```

The GLM Procedure

Class Level Information

Class	Levels	Values
floor	2	H L
feeder	2	H L

Number of observations 40

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	170.4000000	56.8000000	3.68	0.0207
Error	36	555.6000000	15.4333333		
Corrected Total	39	726.0000000			

R-Square	Coeff Var	Root MSE	y Mean
0.234711	6.773324	3.928528	58.00000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
floor	1	144.4000000	144.4000000	9.36	0.0042
feeder	1	6.4000000	6.4000000	0.41	0.5237
floor*feeder	1	19.6000000	19.6000000	1.27	0.2672

Source	DF	Type III SS	Mean Square	F Value	Pr > F
floor	1	144.4000000	144.4000000	9.36	0.0042
feeder	1	6.4000000	6.4000000	0.41	0.5237
floor*feeder	1	19.6000000	19.6000000	1.27	0.2672

Least Squares Means

floor	y LSMEAN
H	59.9000000
L	56.1000000

feeder	y LSMEAN
H	58.4000000
L	57.6000000

floor	feeder	y LSMEAN
H	H	61.0000000
H	L	58.8000000
L	H	55.8000000
L	L	56.4000000

Parameter	Estimate	Standard Error	t Value	Pr > |t|	95% Confidence Limits	
H floor - L floor	3.80000000	1.24230968	3.06	0.0042	1.28047920	6.31952080
H feeder - L feeder	0.80000000	1.24230968	0.64	0.5237	-1.71952080	3.31952080

```
data two; set one;
  if floor='H' and feeder='L' then y=y-8;
run;

proc glm;
  class floor feeder;
  model y=floor feeder floor*feeder;
  lsmeans floor*feeder / slice=floor;
  lsmeans floor*feeder / slice=feeder;
  estimate 'H floor - L floor when feeder L' floor 1 -1 floor*feeder 0 1  0 -1;
  estimate 'H floor - L floor when feeder H' floor 1 -1 floor*feeder 1 0 -1  0;
  estimate 'H feeder - L feeder when floor L' feeder 1 -1 floor*feeder 0  0  1 -1;
  estimate 'H feeder - L feeder when floor H' feeder 1 -1 floor*feeder 1 -1  0  0;
run;
```

The GLM Procedure

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	522.400000	174.133333	11.28	<.0001
Error	36	555.600000	15.433333		
Corrected Total	39	1078.000000			

R-Square	Coeff Var	Root MSE	y Mean
0.484601	7.015229	3.928528	56.00000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
floor	1	0.4000000	0.4000000	0.03	0.8730
feeder	1	230.4000000	230.4000000	14.93	0.0004
floor*feeder	1	291.6000000	291.6000000	18.89	0.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
floor	1	0.4000000	0.4000000	0.03	0.8730
feeder	1	230.4000000	230.4000000	14.93	0.0004
floor*feeder	1	291.6000000	291.6000000	18.89	0.0001

Least Squares Means

floor	feeder	y LSMEAN
H	H	61.0000000
H	L	50.8000000
L	H	55.8000000
L	L	56.4000000

floor*feeder Effect Sliced by floor for y

floor	DF	Sum of Squares	Mean Square	F Value	Pr > F
H	1	520.200000	520.200000	33.71	<.0001
L	1	1.800000	1.800000	0.12	0.7347

floor*feeder Effect Sliced by feeder for y

feeder	DF	Sum of Squares	Mean Square	F Value	Pr > F
H	1	135.200000	135.200000	8.76	0.0054
L	1	156.800000	156.800000	10.16	0.0030

Parameter	Estimate	Standard Error	t Value	Pr > |t|
H floor - L floor when feeder L	-5.6000000	1.75689119	-3.19	0.0030
H floor - L floor when feeder H	5.2000000	1.75689119	2.96	0.0054
H feeder - L feeder when floor L	-0.6000000	1.75689119	-0.34	0.7347
H feeder - L feeder when floor H	10.2000000	1.75689119	5.81	<.0001