The Sign Test and the Wilcoxon Signed-Rank Test

   Researchers wished to determine if two preparations of a virus would produce different effects on tobacco plants. Twelve tobacco leaves were used in the experiment. Half of each leaf, selected at random, was treated with preparation 1 of the virus. The other half of each leaf was treated with preparation 2. The response variable measured was the number of lesions (small dark, easily counted rings) appearing on the half leaf. The numbers in the table below give the number of lesions per half leaf.

Leaf	1	2	3	4	5	6	7	8	9	10	11	12
Preparation 1	55	25	21	7	16	14	7	9	5	4	4	1
Preparation 2	17	17	14	11	10	9	7	5	3	2	1	0

1. Do the two preparations of the virus produce significantly different effects? First conduct a t-test to answer this question.

2. Are there any reasons why the results of the t-test might be questionable here?

3. Conduct a *sign test* to determine if the two preparations of the virus produce significantly different effects. To conduct a *sign test* you first count K = the number of data pairs with a positive difference. If there is no difference between treatments, K has a mean of $\text{Mean}(K) = n/2$ and a standard deviation of $\text{SD}(K) = \sqrt{n/4}$, where n is the number of data pairs with a non-zero difference. When n is large enough,

$$Z = \frac{K - \text{Mean}(K)}{\text{SD}(K)} = \frac{K - n/2}{\sqrt{n/4}}$$

has a distribution that is approximately normal. The normal approximation works best with $n \geq 20$, but it can give you a rough idea of the real p-value for n as small as 10. It is always best to use the exact p-value provided by, for example, *proc univariate* in SAS.

4. Now answer the previous question using the *Wilcoxon signed-rank test*.

Leaf	1	2	3	4	5	6	7	8	9	10	11	12
Preparation 1	55	25	21	7	16	14	7	9	5	4	4	1
Preparation 2	17	17	14	11	10	9	7	5	3	2	1	0

(a) Compute the difference for each data pair.

(b) Drop zeros from the list.

(c) Order the absolute differences (ignore whether positive or negative) and assign them ranks. (Treat ties just as in the rank-sum test by replacing all tied values with the average of ranks.)

(d) The *signed-rank statistic* is $S =$ the sum of the ranks for the pairs with a positive difference. Compute S for this data.

(e) If there is no difference between the virus preparations, the mean and standard deviation of S are

$$\text{Mean}(S) = n(n+1)/4 \qquad \text{SD}(S) = \sqrt{\sum_{i=1}^{n} R_i^2/4},$$

where n is the number of non-zero differences and $\sum_{i=1}^{n} R_i^2$ is the sum of the squares of all the ranks. (The formula for $\text{SD}(S)$ in the text applies only when there are no ties.) Find $\text{Mean}(S)$ and $\text{SD}(S)$.

(f) If n is not too small and there are not an excessive number of ties, then

$$Z = \frac{S - \text{Mean}(S)}{\text{SD}(S)} = \frac{S - n(n+1)/4}{\sqrt{\sum_{i=1}^{n} R_i^2/4}}$$

is approximately normal. Find the test statistic Z.

(g) Give an approximate two-sided p-value and a conclusion.