The One-Way Analysis of Variance F-Test

Doughnuts absorb various amounts of fat during cooking. An experiment was conducted to determine if amount of fat absorbed depends on the type of fat used. For each of four fats, six batches of doughnuts were prepared. The data in the table below include the amount of fat absorbed in grams for each batch.

Fat Type	Fat Absorbed in grams						Average	Variance
1	65	73	69	78	57	96	73	178.0
2	78	91	97	82	85	77	85	60.4
3	75	93	78	71	63	76	76	97.6
4	55	66	49	64	70	68	62	67.6

1. Make side-by-side stem-and-leaf plots that will allow you to visually compare the fat absorption levels of the four fat types.

2. Based on your visual comparison of the data, do you think the amount of fat absorbed depends on the fat type?

We now will learn how to conduct an F-test to determine if the amount of fat absorbed does vary with fat type. We begin by identifying the assumptions of the test.

Assumptions

- Data are independent simple random samples from each of I populations OR data come from a completely randomized experiment with I treatment groups.
- The ith population or treatment group has a distribution that is approximately normal with mean μ_i and standard deviation σ_i.
- The standard deviations $\sigma_1, \ldots, \sigma_I$ are all approximately equal to a single value σ.

Additional Notation

$$n_i = \text{size of sample } i \qquad \bar{Y}_i = \text{average of sample } i \qquad s_i = \text{standard deviation of sample } i$$

$$n = \sum_{i=1}^{I} n_i = \text{total number of observations} \qquad \bar{Y} = \text{average of all observations}$$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2 + \cdots + (n_I-1)s_I^2}{(n_1-1)+(n_2-1)+\cdots+(n_I-1)}} = \sqrt{\frac{\sum_{i=1}^{I}(n_i-1)s_i^2}{n-I}} = \text{pooled estimate of standard deviation } \sigma$$

3. What are I, n_1, n_3, n, $\bar{Y}_2$, $\bar{Y}_4$, s_2, $\bar{Y}$, μ_3, and σ in the doughnut problem?

4. Does it appear that the data on fat absorption levels could have come from four normal or nearly normal distributions based on the pictures you made for Problem 1?

5. Does it appear that the data for the four groups could have come from four populations which share a common standard deviation σ? (It is often safe to assume that the answer to this question is yes if the largest sample SD divided by the smallest sample SD is less than or equal to 2 – especially when there are an equal number of observations in each group.)

6. To test $H_0 : \mu_1 = \mu_2 = \cdots = \mu_I$ against H_A : not all μ_i are equal, we first compute

$$MSB = \frac{n_1(\bar{Y}_1 - \bar{Y})^2 + n_2(\bar{Y}_2 - \bar{Y})^2 + \cdots + n_I(\bar{Y}_I - \bar{Y})^2}{I-1} = \frac{\sum_{i=1}^{I} n_i(\bar{Y}_i - \bar{Y})^2}{I-1}.$$

   MSB stands for *Mean Square Between* the I groups. In general, *mean square* refers to a sum of squares divided by the appropriate degrees of freedom. In this case MSB provides a measure of how far the I group means $(\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_I)$ are from the mean of all the data $(\bar{Y})$. If the I group means are far from each other, MSB will be large, and we will have some evidence that the null hypothesis H_0 is not true. Compute MSB for the doughnut data.

7. How large does MSB need to be before we will no longer believe H_0 is true? That depends on the natural variation in the data. If σ^2 is large, $\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_I$ might end up being far apart (and MSB large) by chance. Thus we judge the size of MSB relative to our estimate of σ^2. We use s_p^2 to estimate σ^2. Our estimate s_p^2 is sometimes denoted MSW for *Mean Square Within* groups or MSE for *Mean Square Error*. The formula for s_p^2 shows that it is a weighted average of the within-group variances $s_1^2, s_2^2, \ldots, s_I^2$. Each within-group variance is a sum of squares divided by the appropriate degrees of freedom. Thus it makes sense to refer to s_p^2 as a *mean square*. Compute $s_p^2 = MSW = MSE$ for the doughnut data.

8. Our test statistic for testing H_0 vs. H_A is $F = \frac{MSB}{MSW}$. The test is called an F-test, and the statistic is called an F-statistic. The label F was first used by George W. Snedecor in honor of Sir Ronald A. Fisher. Compute the F-statistic for the doughnut data.

9. We determine the p-value for the F-test by comparing the test statistic F to an F-distribution with $I - 1$ and $n - I$ degrees of freedom. Note that $I - 1$ is the degrees of freedom associated with MSB, and $n - I$ is the degrees of freedom associated with MSW. Because MSB is in the numerator (top) and MSW is in the denominator (bottom) of the F-statistic, $I - 1$ and $n - I$ are called numerator and denominator degrees of freedom, respectively. From the F-table in the back of your book, what can you say about the p-value in the doughnut problem?

10. Provide a conclusion for this analysis of the doughnut data.