A Primer on Factorials and Combination Numbers

$n!$ is read "n factorial." It is defined for a positive integer n as

$$n! = (n)(n-1)\cdots(2)(1).$$

$0!$ is defined to be 1. Some examples follow.

$$\begin{aligned}
0! &= 1 \\
1! &= 1 \\
2! &= (2)(1) = 2 \\
3! &= (3)(2)(1) = 6 \\
4! &= (4)(3)(2)(1) = 24 \\
5! &= (5)(4)(3)(2)(1) = 120 \\
6! &= (6)(5)(4)(3)(2)(1) = 720 \\
7! &= (7)(6)(5)(4)(3)(2)(1) = 5040
\end{aligned}$$

$n!$ gives the number of ways you can order n objects. For example, there are $3! = 6$ ways to order the letters A, B, and C.

$$ABC \quad ACB \quad BAC \quad BCA \quad CAB \quad CBA$$

$\binom{n}{k} = C_{n,k}$ is read "n choose k." $\binom{n}{k} = C_{n,k}$ is defined for non-negative integers $n \geq k$ as

$$\frac{n!}{(n-k)!k!} = \frac{(n)(n-1)\cdots(n-k+1)}{(k)(k-1)\cdots(2)(1)}.$$

$\binom{n}{k} = C_{n,k}$ gives the number of ways you can choose k objects from a set of n objects. For example

$$\binom{5}{2} = C_{5,2} = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = \frac{(5)(4)(3)(2)(1)}{(3)(2)(1)(2)(1)} = \frac{(5)(4)}{(2)(1)} = \frac{20}{2} = 10$$

is the number of ways to pick two letters from the 5 letters A, B, C, D, and E.

$$AB \quad AC \quad AD \quad AE \quad BC \quad BD \quad BE \quad CD \quad CE \quad DE$$

(Note that here we are not considering order. AB is the same as BA, for example.)

Q: If an instructor wanted to choose a committee of 3 students from a class of 100 students, how many different committees of three are possible?

ANS:

$$\binom{100}{3} = C_{100,3} = \frac{100!}{(100-3)!3!} = \frac{100!}{97!3!} = \frac{(100)(99)(98)}{(3)(2)(1)} = 161700$$

(On a TI calculator try 100 MATH PRB nCr 3.)