

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.  
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (16 points) Let  $f(x) = -3x^2 + 12x - 9$

(a) Circle the correct option:

The graph of  $f$  (opens up / opens down) and has a highest / lowest point.

(b) Find the vertex of the graph of  $f(x)$ .

$$h = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$$

$$k = f(h) = f(2) = -3 \cdot 2^2 + 12 \cdot 2 - 9 = -12 + 24 - 9 = 3$$

$$(h, k) = (2, 3)$$

(c) Find the x-intercept(s) of  $f(x)$ .

$$0 = f(x) = -3(x-2)^2 + 3$$

$$-3 = -3(x-2)^2$$

$$1 = (x-2)^2$$

$$\pm 1 = x - 2$$

$$x = 2 \pm 1$$

$$x = 1 \text{ or } 3$$

(d) Find the y-intercept of  $f(x)$

$$f(0) = -3(0)^2 + 12 \cdot 0 - 9 = -9$$

$$y = -9$$

## Test #2

2. (10 points) Identify the domain of each of the following functions.

(a)  $f(x) = x^{15} + 25x^5 + x^3$

All real numbers

(b)  $R(x) = \frac{x^2-4}{x^2-x-2} = \frac{x^2-4}{(x-2)(x+1)}$

$\text{dom } R = \{x \mid x \neq 2, x \neq -1\}$

(c)  $s(x) = \frac{1}{\sqrt{x-4}}$

$x-4 > 0 \Rightarrow x > 4$

$\text{dom } s = \{x \mid x > 4\}$

3. (16 points) The price  $p$  (in dollars) and the quantity  $x$  sold of a certain product obey the demand equation  $p = -\frac{1}{10}x + 150$ .

(a) Express the revenue  $R$  as a function of  $x$ .

$$R(x) = px = -\frac{1}{10}x^2 + 150x$$

(b) What is the revenue (in dollars) if 10 units are sold?

$$R(10) = -\frac{1}{10}10^2 + 150 \cdot 10 = -10 + 1500 = 1490$$

$$\boxed{\$1490}$$

(c) What quantity  $x$  maximizes the revenue?

$$h = \frac{-b}{2a} = \frac{-150}{2 \cdot -\frac{1}{10}} = \frac{-150}{-2/10} = \frac{150}{1/5} = \boxed{750}$$

## Test #2

4. (8 points) Determine whether each of the following functions is a polynomial. For those which are polynomials, state the degree.

(a)  $f(x) = x^5 + 6x + 6$

Polynomial - degree 5

(b)  $g(x) = x^{\frac{3}{2}} + 4x + 2$

Not a polynomial - cannot have fractional exponents

(c)  $h(x) = 15$

Polynomial - degree 0

5. (20 points) Let  $f(x) = x^3 - 3x^2 - 6x + 8$

- (a) Determine how many positive and negative real zeros  $f(x)$  may have.

$$f(-x) = -x^3 - 3x^2 + 6x + 8$$

$f$  has 2 or 0 positive real zeros, and  
1 negative real zero.

- (b) List the possible rational roots of  $f(x)$ .

$$p: \pm 1, \pm 2, \pm 4, \pm 8$$

$$q: \pm 1$$

$$\boxed{\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8}$$

- (c) Factor  $f(x)$  completely over the real numbers.

$$f(1) = 1 - 3 - 6 + 8 = 0$$

$$\begin{array}{r} 1 \overline{) 1 - 3 - 6 \ 8} \\ \underline{1 \quad - 2 \quad - 8} \\ 1 \quad - 2 \quad - 8 \quad 0 \end{array}$$

$$f(x) = (x-1)(x^2 - 2x - 8)$$

$$\boxed{f(x) = (x-1)(x-4)(x+2)}$$

either factor or use the quadratic formula on  $x^2 - 2x - 8$  to find  $x^2 - 2x - 8 = (x-4)(x+2)$

## Test #2

6. (20 points) Let  $f(x) = (x - 2)(x^2 + 4x + 4)$

(a) List the real zeros of  $f(x)$  and their multiplicities.

$$x^2 + 4x + 4 = (x + 2)^2$$

$$f(x) = (x - 2)(x + 2)^2$$

2 of multiplicity 1

-2 of multiplicity 2

(b) Identify the y-intercept of the graph of  $f$ .

$$f(0) = (0 - 2)(0^2 + 4 \cdot 0 + 4) = (-2)(4) = -8$$

$$\boxed{y = -8}$$

(c) Determine whether the graph crosses or touches the x-axis at each x-intercept.  
(e.g. "The graph of  $f(x)$  (crosses/touches) the x-axis at  $x=c$ .")

The graph of  $f$  crosses the x-axis at  $x = 2$   
and touches the x-axis at  $x = -2$

(d) What power function  $g(x)$  does the graph of  $f$  resemble for large values of  $|x|$ ?

$$g(x) = x^3$$

7. (10 points) Let  $G(x) = \frac{x-4}{x^2-4} = \frac{x-4}{(x+2)(x-2)}$

(a) Circle the correct option:

$G(x)$  is / is not) in lowest terms.

$G(x)$  is a proper / improper) rational function.

## Test #2

- (b) List the vertical asymptote(s) of  $G(x)$ .

Vertical asymptotes are at the zeros of the denominator of  $G(x)$

i.e.  $x=2$  and  $x=-2$

- (c) List the horizontal asymptote(s) of  $G(x)$ .

$G$  is proper, so  $y=0$  is a horizontal asymptote of  $G(x)$

8. (5 points) EXTRA CREDIT. Write a quadratic function  $f(x)$  which has vertex  $(2, 4)$  and y-intercept  $(0, 2)$ .

$$f(x) = a(x-h)^2 + k$$

$$= a(x-2)^2 + 4$$

$$2 = f(0) = a(0-2)^2 + 4$$

$$= a(-2)^2 + 4$$

$$= 4a + 4$$

$$2 = 4a + 4$$

$$-2 = 4a$$

$$a = \frac{-2}{4} = -\frac{1}{2}$$

$$f(x) = -\frac{1}{2}(x-2)^2 + 4$$