MATH 140, Section K2
October 30, 2009

Quiz #8

Name: Key

READ AND FOLLOW ALL DIRECTIONS. CIRCLE YOUR FINAL ANSWERS.
SHOW ALL WORK TO RECEIVE FULL CREDIT. NO CALCULATORS.

1. (4 points) Let $f(x) = \frac{1}{x-2}$ and $g(x) = \frac{2x^2}{x+4}$

   (a) Find $(f \circ g)(4)$.

$$(f \circ g)(4) = f(g(4)) = f\left(\frac{2 \cdot 4^2}{4+4}\right) = f\left(\frac{2 \cdot 16}{2 \cdot 4}\right) = f(4) = \frac{1}{4-2} = \frac{1}{2}$$

   (b) Find $(g \circ f)(3)$.

$$(g \circ f)(3) = g(f(3)) = g\left(\frac{1}{3-2}\right) = g(1) = \frac{2 \cdot 1^2}{1+4} = \frac{2}{5}$$

2. (4 points) Verify that the functions $f(x) = 3x + 4$ and $g(x) = \frac{1}{3}(x - 4)$ are inverses of each other by showing that $f \circ g(x) = x$ and $g \circ f(x) = x$.

$$f(g(x)) = 3\left(\tfrac{1}{3}(x-4)\right) + 4$$
$$= x - 4 + 4$$
$$= x$$

$$g(f(x)) = \tfrac{1}{3}\left((3x+4) - 4\right)$$
$$= \tfrac{1}{3}(3x) = x$$

So we conclude $g(x) = f^{-1}(x)$

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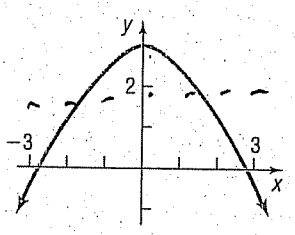
3. (4 points) Determine whether or not each of the given functions is one-to-one. Explain your reasoning.

   (a) $\{(-2,6);(-1,3);(0,2);(1,5);(2,8)\}$ one-to-one

   Each input gives a distinct output (i.e. no two inputs have the same output)

   (b)



   Not one-to-one

   the graph of this function fails the horizontal line test

4. (8 points) The function $f(x) = \frac{2x}{x-6}$ is one-to-one. Find its inverse function $f^{-1}(x)$. For extra credit, verify $f^{-1}(f(x)) = x$.

$x = f(f^{-1}(x)) = \frac{2f^{-1}(x)}{f^{-1}(x) - 6}$

$x(f^{-1}(x) - 6) = 2f^{-1}(x)$

$xf^{-1}(x) - 6x = 2f^{-1}(x)$

$+6x \quad +6x$

$-2f^{-1}(x) \quad -2f^{-1}(x)$

$xf^{-1}(x) - 2f^{-1}(x) = 6x$

$f^{-1}(x) \cdot (x-2) = 6x$

$$\boxed{f^{-1}(x) = \frac{6x}{x-2}}$$

XC

$$f^{-1}(f(x)) = \frac{6\left(\frac{2x}{x-6}\right)}{\frac{2x}{x-6} - 2\left(\frac{x-6}{x-6}\right)}$$

$$= \frac{\frac{12x}{x-6}}{\frac{2x-2x+12}{x-6}}$$

$$= \frac{12x}{x-6} \cdot \frac{x-6}{12}$$

$$= \frac{12x}{12} = x$$