1 MATH 317, Fall 2007, Test 3

Problem 1 (25 points) Consider a linear transformation $L_1 : \mathcal{V} \to \mathcal{W}$ and a linear transformation $L_2 : \mathcal{W} \to \mathcal{X}$, with $\mathcal{V}$, $\mathcal{W}$ and $\mathcal{X}$, vector spaces.

a) Show $\ker(L_1) \subseteq \ker(L_2 \circ L_1)$.

b) Show $range(L_2 \circ L_1) \subseteq range(L_2)$.

Problem 2 (15 points) Consider the linear transformation $L : \mathcal{P}_2 \to \mathcal{P}_2$ defined by

$$L(\mathbf{p}(x)) = \mathbf{p}(x) + \frac{d}{dx}\mathbf{p}(x).$$

Consider the basis in $\mathcal{P}_2$, $B = (1, 1 + x, 1 - x^2)$. Calculate the matrix associated with L, with respect to this basis, A_{BB}.

Problem 3 (15 points) Consider a linear transformation $L : \mathcal{P}_2 \to \mathcal{P}_2$ whose matrix with respect to the basis $B = (1, 1 + x, -x^2)$ is

$$A_{BB} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Find the matrix for the same linear transformation with respect to the standard basis $S = (1, x, x^2)$, i.e., A_{SS}.

Problem 4 (15 points) Consider the 2×2 matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

and the linear transformation on the set of 2×2 matrices $\mathcal{M}_{2,2}$, $L : \mathcal{M}_{2,2} \to \mathcal{M}_{2,2}$ defined by

$$L(M) := AM.$$

a) Find the range of this linear transformation.

b) Find the Kernel of this linear transformation.

c) Verify the dimension theorem.

d) Is this Linear transformation an isomorphism? Why or why not?

Problem 5 (15 points) Consider the same linear transformation as in Problem 4.

a) Find the eigenvalues of this linear transformation.

b) Find the corresponding eigenspaces.

c) Is this linear operator diagonalizable? Why or why not?

Problem 6 (15 points) Consider the orthogonal set in R^3,

$$\{\vec{w}_1, \vec{w}_2\}, \qquad \vec{w}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \vec{w}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

Using Gram-Schmidt process, find 1 more vector $\vec{w}_3$, so that $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is an orthogonal set spanning R^3.