

1 MATH 317, Fall 2007, Test 2

Problem 1 (25 points) Consider a subset S of a vector space \mathcal{V} and an element \vec{v} of \mathcal{V} . Prove that

$$\text{span}(S) = \text{span}(S \cup \{\vec{v}\}),$$

if and only if

$$\vec{v} \in \text{span}(S).$$

Problem 2 (15 points) Let C be the matrix

$$C := \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 4 & 3 \end{pmatrix},$$

and A and B be matrices such that

$$\det(A) = 4, \det(B) = 8.$$

Calculate $\det(2A^{-1}B^TC)$.

Problem 3 (15 points) Diagonalize if possible the matrix

$$A := \begin{pmatrix} 4 & 1 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

If not possible explain why.

Problem 4 (15 points) Consider the vector space of polynomials of degree ≤ 3 , \mathcal{P}_3 , and the subset $S \subseteq \mathcal{P}_3$,

$$S = \{1, 1 + x^3, 3 + 2x^3, 1 + x^2 + 2x\}$$

- Is S a subspace of \mathcal{P}_3 ?
- Find two different bases for $\text{span}(S)$.
- What is the dimension of $\text{span}(S)$?
- If necessary, complete a basis of $\text{span}(S)$ to find a basis of \mathcal{P}_3 .

Problem 5 (15 points) Consider the vector space of 3×2 matrices with the ordered basis

$$B := \left(\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 3 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \end{pmatrix} \right).$$

Consider the matrix

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \\ 0 & 1 \end{pmatrix},$$

and calculate the coordinatization $[A]_B$ of A with respect to the basis B .

Problem 6 (15 points) Consider the vector space of 2×2 symmetric matrices and the standard basis

$$S = \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right)$$

Consider the basis

$$B = \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right)$$

and the basis

$$C = \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} \right).$$

Calculate the transition matrix from basis B to basis C , P_{BC} with two methods 1) Directly without using the standard basis S 2) Using the standard basis S .