1 MATH 317, Fall 2007, Test 1

Problem 1 (25 points) Given two square upper triangular matrices $A$ and $B$, prove that the product $AB$ is also upper triangular.

Problem 2 (15 points) Find the reduced row echelon form of the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 1 & 4 \end{pmatrix}. $$

You are required to show your work.

Problem 3 (15 points) Consider a linear system of equations $AX = B$ and assume that after Gauss reduction the augmented matrix associated with this system is

$$[A|B] = \begin{pmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. $$

a) What is the column rank of this matrix? Give a maximum set of linearly independent columns.

b) Describe the set of all the solutions of this system. Give two examples of solution.

Problem 4 (15 points) Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 3 & x \end{pmatrix},$$

with the free parameter $x$. Describe how the rank of $A$ changes with the value of $x$.

Problem 5 (15 points) Given the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix},$$

use row reduction to calculate $A^{-1}$ if it exists. Calculate $(A^2)^{-1}$.

Problem 6 (15 points) Consider a $2 \times 2$ elementary matrix $R$ which upon left multiplication multiplies the 2–nd row of a matrix $A$ by 3, it adds the result to the first row and places the result in the first row, and leaves the remaining row unchanged, i.e., it does the type II operation $3(2) + (1) \rightarrow (1)$.

a) Write down the matrix $R$. 
b) Write \( R \) as a sum of a symmetric matrix and a skew-symmetric matrix.

c) Give a 2 \( \times \) 2 matrix which does the same operation 3(2) + (1) \( \rightarrow \) (1) on columns by right multiplication \( A \rightarrow AR \).