

# 1 MATH 317, Fall 2007, Test 1

**Problem 1** (25 points) Given two square upper triangular matrices  $A$  and  $B$ , prove that the product  $AB$  is also upper triangular.

**Problem 2** (15 points) Find the reduced row echelon form of the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 1 & 4 \end{pmatrix}.$$

You are required to show your work.

**Problem 3** (15 points) Consider a linear system of equations  $AX = B$  and assume that after Gauss reduction the augmented matrix associated with this system is

$$[A|B] = \begin{pmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

- What is the column rank of this matrix? Give a maximum set of linearly independent columns.
- Describe the set of all the solutions of this system. Give two examples of solution.

**Problem 4** (15 points) Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 3 & x \end{pmatrix},$$

with the free parameter  $x$ . Describe how the rank of  $A$  changes with the value of  $x$ .

**Problem 5** (15 points) Given the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix},$$

use row reduction to calculate  $A^{-1}$  if it exists. Calculate  $(A^2)^{-1}$ .

**Problem 6** (15 points) Consider a  $2 \times 2$  elementary matrix  $R$  which upon left multiplication multiplies the 2–nd row of a matrix  $A$  by 3, it adds the result to the first row and places the result in the first row, and leaves the remaining row unchanged, i.e., it does the type  $II$  operation  $3(2) + (1) \rightarrow (1)$ .

- Write down the matrix  $R$ .

b) Write  $R$  as a sum of a symmetric matrix and a skew-symmetric matrix.

c) Give a  $2 \times 2$  matrix which does the same operation  $3(2) + (1) \rightarrow (1)$  on *columns* by right multiplication  $A \rightarrow AR$ .