Test # 2 Solutions.

Pr 1 $\det(A) = 1 \cdot \det\begin{pmatrix} 2 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} =$

$(-1)^{2+1} \det\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} = (-1)(-2) = 2$

Pr 2

$\det\left(\frac{1}{4} C C^T D\right) = \left(\frac{1}{4}\right)^3 \det(C)\det(C^T)\det(D).$

$= \left(\frac{1}{4}\right)^3 [\det(C)]^2 (-1)\det(C) = (-1) \frac{2^3}{4^3} = -\frac{1}{8}$

Pr 3

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \overset{-1}{\sim} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$\det = (-1)(-1) = 1$

Pr 4

$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ $A_1 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$A_2 = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ $x_1 = \dfrac{\det A_1}{\det A}$ $x_2 = \dfrac{\det A_2}{\det A}$

$x_1 = \dfrac{1}{-1} = -1$ $x_2 = \dfrac{-4}{-1} = 4$

Pr 5 Yes it is a vector space

because

(1) It is a subset of the vector space of 2×2 matrices.

(2) It is non empty.

(3) It is closed under sum and multiplication by a scalar.

It is therefore a subspace

Pr 6

$$\begin{pmatrix} 2 & 0 & 2 & 2 \\ 1 & 1 & 0 & 2 \\ 3 & 4 & -1 & 7 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 2 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 4 & -4 & 4 \end{pmatrix} \sim \begin{array}{c} \begin{array}{cccc} p & p & & \end{array} \\ \begin{pmatrix} 2 & 0 & 2 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Basis $\left\{ \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix} \right\}$

rank $= 2$ $\quad$ dim $Nul(A) = 4 - 2 = 2$

Pr 7 $\quad \{ 1+t,\ 1-2t,\ t^2-1 \}$

Pr 8

$$a \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$$

$\Rightarrow a = 0 \quad b = 3 \quad c = 3$

$[A]_B = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$

Pr 8

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$P_{BC} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$[x]_C = P_{BC}[x]_B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ -7 \end{bmatrix}$$

Pr 10 Fix basis $B = (1, t, t^2) = C$

$T(1) = -2$

$T(t) = -2t$

$T(t^2) = 0$

$$A_{BC} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Nul } A_{BC} = \text{span}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \ker(T) = \text{span}\{t^2\}$$

Pr 11

Fix basis $B = C = \left\{ \underset{A_1}{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}, \underset{A_2}{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}, \underset{A_3}{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}, \underset{A_4}{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}} \right\}$

$T(A_1) = A_1$

$T(A_2) = A_3$

$T(A_3) = A_2$

$T(A_4) = A_4$

$$A_{BC} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$Col(A_{BC}) = \mathbb{R}^4 \Rightarrow$ range $T = M_{2,2}$