MATH 307 Test 1 Solutions

Problem 1

a) $[A|b] = \left[\begin{array}{ccc|c} 3 & 2 & -1 & 1 \\ 6 & 4 & 1 & 2 \end{array}\right] \sim$

$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & 1 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

$$3x_1 + 2x_2 - x_3 = 1$$

$$3x_3 = 0 \Rightarrow x_3 = 0$$

$$x_2 = h$$

$$3x_1 + 2h = 1 \Rightarrow x_1 = \frac{1-2h}{3}$$

b)

$$\vec{x} = \begin{pmatrix} \frac{1}{3} \\ 0 \\ 0 \end{pmatrix} + h \begin{pmatrix} -\frac{2}{3} \\ 1 \\ 0 \end{pmatrix}$$

Problem 2

All solutions are given by:

$$\vec{x} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + h \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

for $h = 2$ we get $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ so this is a solution.

Problem 3.

System $A\vec{x} = \vec{b}$

with $A = \begin{pmatrix} 3 & 1 \\ 0 & -2 \\ 4 & 2 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$

should have a solution.

$$\begin{pmatrix} 3 & 1 & 1 \\ 0 & -2 & 4 \\ 4 & 2 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 3 & 1 & 1 \\ 0 & -2 & 4 \\ 0 & \frac{2}{3} & -\frac{4}{3} \end{pmatrix} \sim \begin{pmatrix} 3 & 1 & 1 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

Yes it has solution because no row of the type $(0\ 0 \ldots 0\ *)$ exists. So $\vec{v}$ is in span of $\left\{ \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \right\}$

Problem 4

$$\begin{pmatrix} A_{11} & 8A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{pmatrix} \begin{pmatrix} -2A_{11}^{-1} \\ A_{22} \end{pmatrix} = \begin{pmatrix} -2I + 8I \\ I \end{pmatrix} =$$

$$\begin{pmatrix} 7I \\ I \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 0 & 7 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Problem 5

No they don't. If a) is true then b) is true because

$\vec{v}_1 \in \operatorname{span}\{\vec{v}_2, \vec{v}_3\}$ means

$$\vec{v}_1 = \lambda \vec{v}_2 + \mu \vec{v}_3$$

which means that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly dependent.

However b) $\not\Rightarrow$ a) in general.

It is possible that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly dependent but $\vec{v}_1 \notin \operatorname{span}\{\vec{v}_2, \vec{v}_3\}$

For example $\vec{v}_2 = \vec{v}_3 = \vec{0}$ $\vec{v}_1 \neq \vec{0}$

(5)

Problem 6   No in general it is not ~~linear~~ linear. To be linear we need.

$$T(\vec{x}_1+\vec{x}_2)=T(\vec{x}_1)+T(\vec{x}_2)$$

We have

$*$ $T(\vec{x}_1+\vec{x}_2)= \cancel{A\vec{x}_1+\vec{b}+A\vec{x}_2+\vec{b}} =$

$$A(\vec{x}_1+\vec{x}_2)+\vec{b} = A\vec{x}_1+A\vec{x}_2+\vec{b}$$

$**$ $T(\vec{x}_1)+T(\vec{x}_2)= A\vec{x}_1+\vec{b}+A\vec{x}_2+\vec{b} = A\vec{x}_1+A\vec{x}_2+$

$*$ and $**$ are not the same unless $\vec{b}=2\vec{b}$

that is $\vec{b}=\vec{0}$

Problem 7

$$T(\vec{e}_1)=\begin{pmatrix}1\\1\\1\end{pmatrix} \quad T(\vec{e}_2)=\begin{pmatrix}1\\-1\\-2\end{pmatrix} \Rightarrow A=\begin{pmatrix}1&1\\1&-1\\1&-2\end{pmatrix}$$

Problem 8

onto

$$\begin{pmatrix} 1 & 4 & 8 \\ 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 8 \\ 0 & -3 & -7 \end{pmatrix} \quad (1)$$

Yes every row has a pivot.

one to one

No the system $A\vec{x} = \vec{0}$ has a non trivial solution [(1) has a nonpivot column (the last one)].

Problem 9

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -5 & -3 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{3}{5} & -\frac{1}{5} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -\frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{3}{5} & -\frac{1}{5} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -\frac{1}{5} & \frac{2}{5} \\ \frac{3}{5} & -\frac{1}{5} \end{pmatrix}$$

<u>Problem 10</u>

$$A = \begin{pmatrix} 10 & 28 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

$$10 + 28 = \boxed{38}$$