Practice Midterm Solutions

Pr 1

$$M_y = N_x \ ?$$

$M_y = 0 \quad N_x = 0 \quad$ Yes it is exact.

Pr 2 an integrating factor $\mu = \mu(x,y)$ is a function of x and y such that the equation

$$(*) \quad \mu M + \mu N \frac{dy}{dx} = 0$$

is exact (even though $M + N \frac{dy}{dx} = 0$ is not). If $(*)$ is exact then we can find a function ψ so that

$$\frac{\partial \psi}{\partial x} = \mu M \qquad \frac{\partial \psi}{\partial y} = \mu N$$

The equation $\varphi(x,y) = \text{constant}$ 1P

give the solution implicitly.

Pr 3 a fundamental set is a set of functions

$y_1 \cdots y_n$ such that <u>every</u>

solution y can be written as linear combination of $y_1 - y_n$ i.e.

$$y = C_1 y_1 + C_2 y_2 + \cdots + C_n y_n$$

Pr 4

$$W(t) = \begin{vmatrix} e^t & e^{2t} & e^{-t} \\ e^t & 2e^{2t} & -e^{-t} \\ e^t & 4e^{2t} & e^{-t} \end{vmatrix}$$

$$W(0) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 4 & 1 \end{vmatrix}$$

$$\det(W(0)) = \left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 1 & 2 \\ 1 & 4 & 1 & 1 & 4 \end{array}\right|$$

$$2 - 1 + 4 - 2 + 4 - 1 = 6$$

Pr 5

Guess $y = (At + B)e^t$

$$y' = Ae^t + (At + B)e^t = (At + A + B)e^t$$

$$y'' = (At + (2A + B))e^t$$

$$y''' = (At + (3A + B))e^t$$

$$y''' + y = te^t$$

$$(At + (3A+B))e^t + (At+B)e^t = te^t$$

$$2A = 1 \qquad A = \frac{1}{2}$$

$$3A + 2B = 0 \qquad B = -\frac{3}{4}$$

$$y = \left(\frac{t}{2} - \frac{3}{4}\right) e^t$$

Pr 6 Notice

$$(r+1)^2 = 0 \qquad r_{1/2} = -1$$

$$r^2 + 2r + 1 = 0$$

$$\boxed{y'' + 2y' + y = 0}$$

Pr 7

$$3e^{i\frac{\pi}{4}} = 3\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right] = \frac{3\sqrt{2}}{2} + i\,\frac{3\sqrt{2}}{2}$$