

**MATH 267 Sections A3-C1 List of problems for Final Exams and
Review exercises**

1. Solution of first order separable equations (Section 2.2)
2. Solution of first order linear equations with the method of variation of parameters (Section 2.4).
3. Calculation of Transfer function, transient term and steady term for forced harmonic motion (Section 4.7)
4. Use of Laplace transform to solve differential equations (Section 5.4)
5. Laplace transforms of discontinuous functions (Section 5.5)
6. Inverse Laplace transforms leading to discontinuous functions (Section 5.6)
7. Properties of delta functions and impulse response (Section 5.6)
8. Solutions of linear systems of equations, homogeneous with constant coefficients (Sections 9.1, 9.2, 9.4 and Lecture notes)
9. Calculation of the exponential of a matrix (Sections 9.5 and 9.8)
10. Solution of inhomogeneous linear systems (Section 9.8)
11. Qualitative analysis of planar linear systems (Section 9.3).

Sample problems to be solved in class during the review

1. Find the general solution of the following first order differential equation.

$$\frac{dy}{dx} = \frac{1}{y^2(1+x^2)}.$$

Find the solution corresponding to the initial condition

$$y(1) = 1.$$

2. Use the method of variation of parameters to solve the following linear, first order, differential equation with the given initial condition

$$y' = ty + e^{\frac{1}{2}t^2} \cos(t), \quad y(0) = 1.$$

3. Consider a circuit driven by a sinusoidal voltage and modeled by the differential equation

$$y'' + 4y' + 3y = \cos(\omega t + \psi)$$

- Calculate the Transfer Function $H(i\omega)$ associated to this equation.
- Calculate the Gain $G(\omega)$ and the Phase $\phi(\omega)$, namely, write $H(i\omega)$ as

$$H(i\omega) = G(\omega)e^{-i\phi(\omega)}.$$

- Assume a forcing voltage $\cos(\omega t + \psi) = \cos(2t + \frac{\pi}{3})$ and the initial condition

$$y(0) = y'(0) = 1.$$

Using the Transfer function, calculate the Steady State Term and the Transient Term for the solution $y = y(t)$.

4. Use the Laplace transform to solve the following initial value problem

$$y^{(4)} - y = e^t, \quad y(0) = y'(0) = y''(0) = y'''(0) = 0.$$

5. Calculate the Laplace transform of the following discontinuous function $g(t)$ by first expressing it in terms of Heaviside function

$$g(t) = t^2 e^t, \quad 0 \leq t < 1,$$

$$g(t) = 2, \quad 1 \leq t < \infty$$

6. Calculate the inverse Laplace transform of the following function

$$F(s) = \frac{e^{-2s}}{(s^2 + 1)(s + 4)}.$$

7. • Calculate

$$\int_0^2 \cos(t) \delta_{\frac{\pi}{4}}(t) dt,$$

and

$$\int_0^{\frac{\pi}{6}} \cos(t) \delta_{\frac{\pi}{4}}(t) dt.$$

- Using Laplace transform, calculate the solution of the following initial value problem:

$$y'' + y = \delta_0(t) + 1, \quad y(0) = y'(0) = 1.$$

8. Consider the linear homogeneous equation

$$\vec{x} = A\vec{x},$$

with A given by

$$A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}.$$

Sketch the phase plot and describe the nature of the equilibrium points.