Summary of More Recent Research

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My current area of research is in *control and analysis of quantum mechanical systems.* In this study, I have applied and enhanced tools of *geometric control theory* and investigated models and mathematical problems in *quantum information theory.* Previous research has focused on general nonlinear control theory, optimal control of linear and nonlinear systems and applications of ergodic theory to the control and analysis of fluid mixing. In this summary, I will not consider contributions in these previous areas of research as I will focus on more recent research. Numbered references refer to publications listed in the 'Publications' section. References denoted by a letter are listed in the references section.

Methodologies for quantum control based on decompositions of Lie groups

Decompositions of Lie groups are of great interest in the theory of quantum control and quantum information. They allow one to decompose the task of controlling the state of a quantum system into simpler sub-tasks. These decompositions also reveal several features of quantum dynamics concerning entanglement and locality. Moreover they are the starting point for the design of the so-called *quantum circuits*, that is, cascades of unitary quantum evolutions from a certain set (quantum gates), which perform prescribed transformations of the quantum state (computations) (see, e.g., [A]). One of the first studies in this direction was [24] where I provided a very explicit method for the control of a spin $\frac{1}{2}$ particle and two interacting spin $\frac{1}{2}$ particles. This method of design can take into account an arbitrary bound on the control. Other algorithms for control which use Lie group decompositions are given in [19], [28]. For the case of the Lie group $SU(2)$, which is the Lie group of interest in the analysis of a spin $\frac{1}{2}$, I have provided a switching control algorithm in [21] which achieves the target with the minimum number of switches. These results are of general interest in the theory of Lie groups. In [B], [C] and my previous work [26] the problem of *uniform finite generation* of Lie groups was considered. Given a set of generators $\mathcal{A}$ of a Lie algebra $\mathcal{L}$, the Lie group associated with $\mathcal{L}$, is said to be uniformly finitely generated by $\mathcal{A}$, if every element X of the Lie group can be written as $X = \prod_{j=1}^{N} e^{F_j t_j}$ with $F_j \in \mathcal{A}$, $t_j \in \mathbf{R}$ and the number N is uniformly bounded as X varies in the Lie group. The maximum value of N is called the *order of generation* of the Lie group with respect to $\mathcal{A}$. The results of [21] go beyond what was known before in that, given a set of generators and a value X in the Lie group $SU(2)$ (or $SO(3)$) the exact minimum number of factors is explicitly calculated *as a function of* X.

When dealing with multipartite quantum systems, it is natural to look for decompositions of the Lie group of evolutions, $U(n)$, in terms of evolutions on the single quantum subsystems. A first study given in [D] proposed the *Concurrence Canonical Decomposition* (CCD). The concurrence is a measure of quantum entanglement, that is, a function of the quantum state, satisfying certain axioms [E] which measures the quantity of entanglement of the state. The CCD is a way to decompose the evolution of the quantum state in a part which modifies the concurrence (and therefore entanglement) and a part that does not. It applies to systems of N qubits (two level quantum systems) and it is a tool to study entanglement dynamics. In [11], I have greatly generalized the CCD introducing a new type of decomposition which I called the *Odd-Even Decomposition* (OED). The OED is a way to decompose the dynamics of a general multipartite quantum system, with arbitrary dimensions, using Cartan decompositions [F], defined for the single subsystems. The CCD is obtained as a special case when all the subsystems have dimension 2 and only a special type of Cartan decomposition is performed on each subsystem. As discussed in my book [1], the OED decomposition is related to *generalized concurrences*, introduced in [G], which have a role in the analysis and detection of entanglement. The paper [11] contains several further results. It establishes a decomposition of the dynamics of quantum systems in terms of time symmetries, generalizing the results of [H] for spin $\frac{1}{2}$'s to general spins and it links *quantum symmetries* [I] to the *Cartan involutions* [F] used in the classification of symmetric spaces.

Applying Lie group decompositions in a recursive manner, one can obtain more useful decompositions. This has led to a large effort by researchers to give algorithms for the *recursive* decomposition of Lie groups and in particular of the unitary group $U(n)$. One such decomposition was given by N. Khaneja and S. Glaser in [J] for a quantum system of N qubits. Another decomposition was given by my R. Romano and myself in [12]. This line of research has lead to the recent paper [9], co-authored by M. Dagli, J.Smith and myself. In this paper, we recognized a link between recursive Lie algebra decompositions and *Lie algebra gradings* and the fact that Lie algebra gradings can be obtained by combining various decompositions. Using these ideas, it is possible to design a virtually unbounded number of algorithms for recursive decompositions and I showed that the papers [J] and [12] are special cases of this general procedure. In constructing recursive decompositions one uses several types of decompositions and, in that, the OED above discussed is a useful tool. In fact, an example of a new algorithm for the design of recursive decompositions using OED was given in [9].

Identification of the dynamics of quantum spin systems

In [20] F. Albertini and I started a research on the identification problem for spin systems. This problem was motivated by early results on the iso-spectrality of spin Hamiltonians in magnetic molecules [K] and may be described as follows. Assume a network of interacting spins is given as, for example, in a molecule. These spins interact with each-other and with an external control electro-magnetic field. Moreover, the total spin in some direction is measured. The model is characterized by several parameters, including the number of spin particles, the value of their spin, the strength of the interaction between the various spins, the strength of the interaction with the control electro-magnetic field, the initial state of the system. Assume, we perform black-box type of experiments by varying the control and measuring the total spin. We would like to know to what extent we can distinguish different models with this type of technique. The mathematical problem is to classify all the models that are indistinguishable with these experiments. The problem was solved in [20] for networks of spin $\frac{1}{2}$ and then this result was extended in [18] by considering the possibility of performing multiple measurements during the experiment and therefore incorporating the back-action of the measurement in the dynamics. The problem was then solved for the case of two spin 1's in [17] and in full generality in [8]. In the latter work, no assumption is made on the number of spin particles nor on their values (which could be different from one another). Moreover in this work a method based on concepts of Lie algebra theory is introduced to tackle this type of equivalence problems for other models. Instrumental to the proof of the main result in [8] is the OED decomposition introduced in [11]. Moreover, the paper [8] contains several proofs for results on subalgebras of the classical Lie algebras which are of independent interest.

Controllability and dynamical analysis

The application of geometric control theory results [L] solved the problem of determining the set of possible evolutions of finite dimensional quantum systems (see, e.g., [Q]). A quantum system is controllable and therefore the set of all the possible evolutions is the whole Lie group $U(n)$ (or $SU(n)$) if the Lie algebra generated by all the possible Hamiltonians of the system is the full $u(n)$ or $su(n)$. More, in general, this Lie algebra, called *the dynamical Lie algebra*, determines the Lie group on which the evolution of the quantum system occurs. Several questions remained however open concerning controllability in terms of state to state transfer. In the paper [23] I defined several physically motivated notions of controllability for quantum systems and gave practical criteria to verify them. These tests involve only linear algebra type of calculations, such as determining the rank and the dimension of the kernel of a linear transformation. One more notion of controllability, *indirect controllability*, was studied by my Raffaele Romano and myself in [13], [16]. Indirect controllability refers to the situation where the Hamiltonian of a quantum system is given and the control occurs via interaction with an auxiliary quantum system (the *probe*) whose initial state can be adjusted (at will).

The knowledge of the dynamical Lie algebra $\mathcal{L}$ not only gives information on the controllability of the quantum system under consideration but determines the very structure of the dynamics of the system. One typically has a set of matrices which are a basis of the Lie algebra $\mathcal{L}$ and would like to explore its nature. Levi's theorem (see, e.g., [M]) implies that $\mathcal{L}$ is the vector space sum of a semisimple Lie algebra and a solvable Lie algebra. Moreover the semisimple part (by definition) can be decomposed into simple Lie algebras. This decomposition determines the structure of the dynamics of the quantum system. Algorithms to calculate such a decomposition exist for general Lie algebras [M] but I have given in [44] simplified algorithms for the special case of interest in quantum control, where $\mathcal{L}$ is a subalgebra of $u(n)$. The analysis of the dynamical Lie algebra provides a powerful approach to the control of quantum systems and several examples of this are given in [44].

Methods to determine the state of quantum systems

One important area of study in quantum mechanics deals with determining or estimating the state of a quantum system through the measurement of a *quorum* of observables. In control theory, one studies to what extent it is possible to determine the initial state of a system by a combined action of dynamics and measurement. Such a property is called *observability*. In [22], I studied the *observability* of quantum systems and gave definitions and tests for it. I showed how quantum dynamics can be in general decomposed in an observable and unobservable part. This study, mostly concerned the standard Von Neumann-Lüders type of measurement of quantum mechanics. However, in [15], R. Romano and I extended it to include more general measurements (cf., e.g., [N]). The paper [15] emphasizes the treatment of *indirect measurement* in which a probe interacts with the system to be measured. One of the results in [15] characterizes the state of the probe which will minimally disturb the system upon measurement. I also gave explicit formulas for the calculation of the quantum state from measurement results and described the design of *observers* for quantum systems. These results on one hand are of fundamental value in the description of structural properties of quantum systems and on the other hand provide a system theoretic alternative to the method of *quantum state tomography* [O] commonly used for quantum state determination. In the fourth chapter of my book [1] I summarized some of these contributions and relate them to quantum state tomographic methods.

Quantum walks

Quantum walks are the quantum counterpart of random walks and are amenable of the same important applications, in particular, as computational tools. Moreover it has been shown (see, e.g., [P]) that in some cases they outperform random walks. In [10] F. Albertini, G. Parlangeli and I started a study of quantum walks on graphs [P] from the point of view of control theory. In the model I considered, the evolution of the walk at each step can change, and can be controlled. This can be obtained with only minor modifications in the existing experimental proposals and it gives several advantages. In [10] I considered the simplest case of a random walk on a cycle and proved two main results: 1) I described the set of all the states that can be achieved for this system; 2) I proved that one of these states is such that the probability is uniformly distributed among the nodes of the associated graph. This is in contrast with the case of standard quantum random walks, where the dynamics is the same at every step. This fact has important applications if, for example, we are trying to use the walk as a random number generator in randomized algorithms. Further results on the controllability of this model were given in [44].

Overall, I believe this area of research is very interesting. In the future I am planning to consider various types of walks, with different graphs, and further explore the interplay between their properties as control systems and their applications as computational tools.

References

[A] M. Mottonen and J.J. Vartiainen, Decomposition of general quantum gates, Ch.7 in *Trends in Quantum Computing Research*, (NOVA Publishers, New York), 2006.
[B] R. Koch and F. Lowenthal, Uniform finite generation of three dimensional linear groups, *Canad. J. Mathematics*, 27, 396-417, 1975.
[C] F. Lowenthal, Uniform finite generation of the rotation group, *Rocky Mountain J. Mathematics*, (1971), 575-586.

[D] S. S. Bullock and G. K. Brennen, Canonical decompositions of n-qubits quantum computations and concurrence, *Journal of Math. Phys.,* 45, No. 6, 2447, (2004).
[E] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press,, Cambridge, U.K., New York, 2000.
[F] S. Helgason, *Differential geometry, Lie groups and symmetric spaces*, Academic Press, New York, 1978.
[G] A. Uhlmann, Fidelity and concurrence of conjugated states, *Phys. Rev. A*, 62, 032307, (2000).
[H] S. S. Bullock, G. K. Brennen and D. P. O'Leary, Time reversal and n-qubit canonical decompositions, *Journal of Math. Phys.,* 46, 062104, (2005)
[I] A. Galindo and P. Pascual, *Quantum Mechanics I*, *Texts and Monographs in Physics*, Springer-Verlag, Heidelberg, 1990.
[J] N. Khaneja and S. J. Glaser, Cartan decomposition of $SU(n)$; constructive controllability of spin systems and universal quantum computing, *Chem. Physics*, 267, 11, 2001.
[K] H-J Schmidt and M. Luban, Continuous families of isospectral Heisenberg spin systems and the limits of inference from measurements, *J. Phys. A: Math. Gen.*, **34**, (2001), 2839-2858.
[L] V. Jurdjević and H. Sussmann, Control systems on Lie groups, *Journal of Differential Equations*, 12, 313-329, (1972).
[M] W. De Graaf, *Lie Algebras; Theory and Algorithms*, North-Holland, 2000.
[N] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*, Oxford University Press, Oxford, New York, 2002.
[O] G. M. D'Ariano, M. G. A. Paris, and M. F. Sacchi, Quantum tomographic methods, *Lect. Notes in Phys.*, 649, 7-58 (2004).
[P] J. Kempe, Quantum random walks - an introductory overview, *Contemporary Physics,* Vol. 44, p. 307-327, (2003).
[Q] G. M. Huang, T. J. Tarn and J. W. Clark, On the controllability of quantum mechanical systems, *Journal of Mathematical Physics*, 24 No. 11, 2608-2618, (1983).

1 Publications

(Several Preprints are Available at http://www.public.iastate.edu/ daless/ or xxx.lanl.gov/quant-ph)

Book

[1] D. D'Alessandro, *Introduction to Quantum Control and Dynamics*, CRC Press, August 2007.

Ph.D. Thesis

[2] D. D'Alessandro, Optimal control techniques with applications (Tecniche di controllo ottimo e loro applicazioni), Ph.D. Thesis, Department of Electrical Engineering, Universita'

degli studi di Padova, Italy, 1996 (in Italian).

[3] D. D'Alessandro, Ergodic Theory and the Control of Mixing in Fluid Flows, Ph. D. Thesis, Department of Mechanical and Environmental Engineering, University of California at Santa Barbara, 1999 (Available at http://www.public.iastate.edu/ daless/).

Book Chapters

[4] F. Albertini and D. D'Alessandro, Remarks on the observability of nonlinear discrete time systems, *System Modeling and Optimization (Prague 1995)*, J. Dolezal & al. eds. 155-162, Chapman & Hall, London, 1996.

[5] D. D'Alessandro and M. Dahleh, Geometric control of quantum mechanical systems, in *Lagrangian and Hamiltonian Methods for Nonlinear Control 2000, (IFAC workshop)*, N.E. Leonard and R. Ortega, eds. 45-49 Pergamon Press, 2000.

[6] D. D'Alessandro, Directions in the theory of quantum control, in *Multidisciplinary Research in Control: The Mohammed Dahleh Legacy* (Santa Barbara CA 2002) 72-80, L. Giarre' and B. Bamieh eds., *Lecture Notes in Control and Information Sciences* 289, Springer Verlag, Berlin, 2003.

[7] D. D'Alessandro and M. Dahleh, Optimal control of two-level quantum systems, Russian translation of journal paper[1], in the volume "Control of Molecular and Quantum Systems" (in Russian: "Upravlenie molekulyarnymi i kvantovymi sistemami"), Ed. A. L. Fradkov, O. A. Yakubovskii, Translated by I. A. Makarov, Moscow-Izhevsk, Institute for Computer Studies, 2003, pp. 245-276.

Journal Papers

[8] F. Albertini and D. D'Alessandro, Analysis and identification of quantum dynamics using Lie algebra homomorphisms and Cartan decompositions, to appear in *SIAM Journal on Control and Optimization*

[9] M. Dagli, D. D'Alessandro and J. Smith, A general framework for recursive decompositions of unitary quantum evolutions, *Journal of Physics A, Mathematical and Theoretical*, **41**, 2008, 155302.

[10] D. D'Alessandro, G. Parlangeli and F. Albertini, Nonstationary quantum walks on the cycle, *J. Phys. A Math. Theor.* (2007) **40**, 14447-14455.

[11] D. D'Alessandro and F. Albertini, Quantum symmetries and Cartan decompositions in arbitrary dimensions, *J. Phys. A Math.* Theor. 40 (2007), no. 10, 2439–2453.

[1] D. D'Alessandro and M. Dahleh, Optimal control of two-level quantum systems, *IEEE Transactions on Automatic Control*, Vol. 46, June 2001, no. 6, pg. 866-876.

[12] D. D'Alessandro and R. Romano, Decomposition of Unitary Evolutions and entanglement dynamics of bipartite quantum systems, *Journal of Mathematical Physics* 47 082109 (2006)

[13] R. Romano and D. D'Alessandro, Environment-mediated control of a quantum system, *Physical Review Letters*, 97, 080402 (2006).

[14] G. Giedke, J. M. Taylor, D. D'Alessandro, M. D. Lukin, A. Imamoglu, Quantum measurement of the nuclear spin polarization in quantum dots, *Physical Review A*, 74 032316 (2006) quant-ph/0508144.

[15] D. D'Alessandro and R. Romano, Further results on the observability of quantum systems under general measurement, *Quantum Information Processing*, Vol. 5, No. 3. (June 2006), pp. 139-160.

[16] R. Romano and D. D'Alessandro, Incoherent control and entanglement for two-dimensional coupled systems, also xxx.lanl.gov, quant-ph/0510020, *Physical Review A* 73 022323 (2006).

[17] D. D'Alessandro, Controllability, observability and parameter identification for two coupled spin 1's,*IEEE Transactions on Automatic Control*, Vol. 50, No. 7, July 2005.

[18] F. Albertini and D. D'Alessandro, Input-Output equivalence of spin networks under multiple measurements, *Mathematics of Control, Signals and Systems*, Vol. 17, No. 1, Pages 1-13, 2005. (Electronic copy available at http://www.springerlink.com/index/10.1007/s00498-004-0146-z)

[19] F. Albertini and D. D'Alessandro, Control of the evolution of Heisenberg spin systems, *European Journal of Control*, Special issue on Lagrangian and Hamiltonian Methods for Nonlinear Control, January 2005.

[20] F. Albertini and D. D'Alessandro, Model identification for spin networks, *Linear Algebra and its Applications* , 394, (2005), 237-256.

[21] D. D'Alessandro, Optimal evaluation of generalized Euler angles with applications to control, *Automatica*, **40** (2004) 1997-2002.

[22] D. D'Alessandro, On quantum state observability and measurement, *Journal of Physics A: Mathematical and General* **36** (2003) 9721-9735

[23] F. Albertini and D. D'Alessandro, Notions of controllability for multilevel bilinear quantum mechanical systems, *IEEE Transactions on Automatic Control*, Vol. 48, No. 8 (2003), pg. 1399-1403.

[24] D. D'Alessandro, Controllability of one spin and two interacting spins, *Mathematics of Control, Signals and Systems*, (2003) 16:1-25.

[25] F. Albertini and D. D'Alessandro, Observability and forward-backward observability of discrete time nonlinear systems, *Mathematics of Control, Signal and Systems,* Vol. 15

(2002), pg. 275-290.

[26] D. D'Alessandro, Uniform finite generation of compact Lie groups, *Systems and Control Letters* **47** (2002) 87-90.

[27] F. Albertini and D. D'Alessandro, The Lie algebra structure and controllability of spin systems, *Linear Algebra and its Applications.* Volume 350, Issues 1-3, 15 July 2002, Pages 213-235.

[28] D. D'Alessandro, The optimal control problem on $SO(4)$ and its applications to quantum control, *IEEE Transactions on Automatic Control*, Vol. 47, No. 1. January 2002.

[29] D. D'Alessandro, I. Mezić and M. Dahleh, Statistical properties of controlled fluid flows with applications to control of mixing, *Systems and Control Letters*, **45** (2002), 249-256.

[30] D. D'Alessandro, Small time controllability of systems on compact Lie groups and spin angular momentum, *Journal of Mathematical Physics*, Vol.42, No. 9, 4488-4496, September 2001.

[31] D. D'Alessandro and M. Dahleh, Optimal control of two-level quantum systems, *IEEE Transactions on Automatic Control*, Vol. 46, June 2001, no. 6, pg. 866-876.

[32] D. D'Alessandro, Topological properties of reachable sets and the control of quantum bits, *Systems and Control Letters*, **41** (2000), pg. 213-221.

[33] F. Albertini, D. D'Alessandro and A. D. B. Paice, Further conditions on the stability of continuous time systems with saturation, *IEEE Trans. on Circuits and Systems I, Fundamental Theory and Applications*, (2000) Vol. 47. No. 10, pp. 723-729.

[34] D. D'Alessandro, M. Dahleh and I. Mezić, Control of mixing in fluid flow: A maximum entropy approach, *IEEE Trans. on Automatic Control*, (1999) Vol. 44. No. 10, pp. 1852-1863.

[35] D. D'Alessandro, Invariant manifolds and projective combinations of solutions of the Riccati differential equations, *Linear Algebra and Its Applications*, 279 (1998) no. 1-3, 181-193.

[36] A. Beghi and D. D'Alessandro, Discrete time optimal control with control-dependent noise and generalized Riccati difference equations, *Automatica*, 34 (1998), no. 8, 1031-4.

[37] D. D'Alessandro, A superposition theorem for solutions of the Riccati difference equation, *Journal of Mathematical Systems Estimation and Control*, 8 (1998), no. 1.

[38] D. D'Alessandro, A parametrization of the nonnegative definite solutions of the algebraic Riccati equation, *Automatica*, 34 (1998), no. 3, 385-388.

[39] D. D'Alessandro, Geometric aspects of the Riccati difference equation in the nonsym-

metric case, *Linear Algebra Appl.*, 255 (1997), 1-18.

[40] M. Pavon and D. D'Alessandro, Families of solutions of matrix Riccati differential equations, *SIAM J. Control Optim.*, 35 (1997), no. 1, 194-204.

[41] D. D'Alessandro and A. Ferrante, Optimal steering for an extended class of nonholonomic systems using Lagrange functionals, *Automatica*, 33 (1997), no. 9, 1635-1646.

[42] F. Albertini and D. D'Alessandro, Asymptotic stability of continuous-time systems with saturation nonlinearities, *Systems Control Lett.*, 29 (1996), no. 3, 175-180.

[43] D. D'Alessandro, On passivity and adaptive stabilization of nonlinear systems, *IEEE Trans. Automat. Control*, 41 (1996), no. 7, 1083-1086.

Papers Submitted to Journals

[44] D. D'Alessandro, Lie Algebraic Analysis and Control of Quantum Dynamics, submitted to *IEEE Transactions on Automatic Control*.

[45] L. Cattaneo and D. D'Alessandro, A note on generalized concurrences and entanglement detection, submitted to *Quantum Information and Computation*.

Refereed Conference Papers

[46] M. Dagli and D. D'Alessandro, Recursive decompositions of quantum dynamics, to appear in Proceedings of Mathematical Theory of Networks and Systems.

[47] F. Albertini and D. D'Alessandro, Lagrangian Formulation and Geometric Control of Switching LC Electrical Networks, In Proceedings of the 45-th conference on Decision and Control to be held in San Diego CA in Dec. 2006.

[48] D. D'Alessandro and R. Romano, Decompositions of unitary evolutions and entanglement dynamics of bipartite quantum systems, In the Proceedings of the 45-th conference on Decision and Control to be held in San Diego CA in Dec. 2006.

[49] R. Romano and D. D'Alessandro, Incoherent controllability and entanglement of quantum systems, In the Proceedings of the 45-th conference on Decision and Control to be held in San Diego CA in Dec. 2006.

[50] D. D'Alessandro and R. Romano, Further results on the observability of quantum mechanical systems under general measurement, in *Proceedings of the 44-th Conference on Decision and Control*, Seville, Spain, Dec. 2005.

[51] D. D'Alessandro, On the observability and state determination of quantum mechanical systems, in *Proceedings of the 43-rd Conference on Decision and Control*, Paradise Island, Bahamas, Dec. 2004.

[52] U. G. Vaidya, D. D'Alessandro and I. Mezić, Control of Heisenberg spin systems; Lie algebraic decompositions and action-angle variables. in *Proceedings of the 42-nd Conference on Decision and Control*

[53] F. Albertini and D. D'Alessandro, Observability, measurement and parameter identification of quantum mechanical systems, in *Proceedings of the 42-nd Conference on Decision and Control*

[54] D. D'Alessandro and V. Dobrovitski, Control of a two level open quantum system, in the Proceedings $41-st$ Conference on Decision and Control, Dec. 2002.

[55] D. D'Alessandro, Contributions of control theory to fundamental quantum mechanics and its applications, in the Proceedings $41-st$ Conference on Decision and Control, Dec. 2002.

[56] F. Borsa, D. D'Alessandro, L. Miller and M. Salapaka, Quantum control of molecular magnets using atomic force microscopy, Proceedings $40-th$ Conference on Decision and Control, Dec. 2001.

[57] F. Albertini and D. D'Alessandro, Notions of controllability for quantum mechanical systems, Proceedings $40-th$ Conference on Decision and Control, Dec. 2001.

[58] F. Albertini and D. D'Alessandro, The Lie algebra structure of spin systems and their controllability properties, in Proceedings $40-th$ Conference on Decision and Control, Dec. 2001.

[59] D. D'Alessandro, Constructive controllability of one and two spin $\frac{1}{2}$ particles, in Proceedings 2001 American Control Conference, Arlington, Virginia, June 2001.

[60] D. D'Alessandro, On the controllability of systems on compact Lie groups and quantum mechanical systems, in *Proceedings 39-th Conference on Decision and Control*, Sydney, Australia, Dec. 2000.

[61] D. D'Alessandro, Algorithms for quantum control based on decompositions of Lie groups, in *Proceedings 39-th Conference on Decision and Control*, Sydney, Australia, Dec. 2000.

[62] D. D'Alessandro and M. Dahleh, Optimal control of two-level quantum systems, *Proceedings American Control Conference*, Chicago, IL, June 2000.

[63] D. D'Alessandro, I. Mezić and M. Dahleh, On the existence of time averages for time varying dynamical systems, in *Proceedings Conference on Decision and Control,* pp. 2065-2070, Tampa (Florida), December 1998.

[64] D. D'Alessandro, M. Dahleh and I. Mezić, Control of fluid mixing using entropy methods, in *Proceedings American Control Conference,* pp. 838-843, Philadelphia (Pennsylvania),

June 1998.

[65] D. D'Alessandro, M. Dahleh and I. Mezić, Maximum entropy approach to the control of mixing, in *Proceedings American Control Conference,* pp. 160-161, Albuquerque (New Mexico), June 1997.

[66] A. Beghi and D. D'Alessandro, Some remarks on FSN models and generalized Riccati equations, in *Proceedings of the 4-th European Control Conference*, paper number 662, Brussels (Belgium), July 1997.

Preprints

[67] D. D'Alessandro, The Lie Algebra Rank Condition for non-bilinear quantum systems, xxx.lanl/quant-ph/0301144.