MATH 265 Section E1, Topics for Final and Practice Final

Topics

1. Domain of functions of two or more variables
2. Level curves and level surfaces
3. Definition of partial derivative
4. Limits of functions of two or more variables
5. Continuity of functions of two or more variables
6. Differentiability of functions of two or more variables
7. Definitions and properties of the gradient
8. Definition and calculation of Directional derivative
9. Chain Rule
10. Calculation of plane normal to a vector and plane tangent to a surface.
11. Definitions of local and global extrema. Methods to solve two-dimensional optimization problems (May omit Lagrange method Section 15.9).
12. Definition of double integrals over rectangle and general regions.
13. Basic properties of double integrals.
14. Simple regions in Cartesian and polar coordinates.
15. Calculation of mass, moments, center of mass.
16. Definition and Calculation of triple integrals.
17. Triple integrals in spherical coordinates
18. Divergence and curl of a vector field. Physical interpretation.
19. Definition and calculation of line integrals of function. Physical interpretation.
20. Definition and calculation of line integrals of vector fields. Physical interpretation.
21. Conservative vector fields and fundamental theorem for line integrals of vector fields.
22. Green's Theorem

23. Definition and calculation of surface integrals of function. Physical interpretation.

24. Definition and calculation of surface integrals of Vector Fields. Physical interpretation.

25. Gauss divergence theorem

26. Stokes Theorem

Practice Final

(17 Questions, 6 points each)

1. Give an example of a function of two variable defined only outside the disk of radius 1, i.e. whose domain is $\{x, y : x^2 + y^2 > 1\}$.

2. Sketch the level curves of the function of two variables
$$z = f(x, y) = x^2 + y^2.$$

3. Consider the function of two variables
$$z = f(x, y) = cos(xy^2) + x.$$
Use the definition of partial derivative to write $f_x(0, \pi)$ as an appropriate limit of a function of one variable.

4. Consider the function $f = f(x, y)$ defined as follows
$$f(x, y) = 1, \qquad (x, y) = (0, 0),$$
$$f(x, y) = e^{-\frac{1}{x^2+y^2}} + 1, \qquad (x, y) \neq (0, 0).$$
Is this function continuous in the whole plane? Justify your answer.

5. Give the definition of Differentiability of a function $f(x, y)$ at a point (x_0, y_0).

6. Find the direction of maximum increase for the function $z = x^2 + y$ at the point $x = 1, y = 1$.

7. Consider a function $f = f(x, y)$, with $x = x(t)$ and $y = y(t)$. Consider the composite function $g(t) = f(x(t), y(t))$ calculate $g'(0)$ if $x'(0) = 1$, $y'(0) = -8$, $\frac{\partial f}{\partial x}(x(0), y(0)) = 9$ and $\frac{\partial f}{\partial y}(x(0), y(0)) = 4$.

8. Assume an electric charge is distributed over the square with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$, with density $x \quad Charge/Area$. Calculate the total Charge.

9. Consider the function $f(x, y)$ defined on the whole plane and assume that, for every value of x and y,

$$1 \leq f(x, y) \leq 2.$$

Give an upper bound and a lower bound for

$$\int \int_A f(x, y) dA,$$

where A is the triangle with vertices $(0, 0)$, $(9, 0)$, $(0, 1)$. Justify your answer.

10. Consider the square with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$, with density $\delta(x, y) = x \qquad Mass/Area$. Find the moment with respect to the y axis.

11. Consider the triple integral

$$\int \int \int_V x^2 + y dV,$$

where V is the upper hemisphere of a ball of radius 2. Write this integral as an iterated integral in spherical coordinates (Do not calculate the integral).

12. A straight wire is plunged into the ocean at an angle 45 degrees with respect to sea level and with a depth 10. The mass of algae which attach to the wire depends on the depth and the rate increases with the depth z as $z^2 Mass/Length$. Calculate the total mass of algae which attach to the wire.

13. Use Green's theorem in the plane to calculate

$$\int_C 2x dx - x dy,$$

where C is the boundary of the square with vertices $(1, 1)$, $(0, 0)$, $(0, 1)$, $(1, 0)$, oriented clock-wise.

14. Consider the surface S given by the portion of $z = 1 - x^2 - y^2$ above the plane $z = 0$. Find a function $f = f(x, y, z)$ which is not identically zero such that the surface integral

$$\int \int_S f(x, y, z) dS$$

is equal to zero.

15. Consider the surface S given by the portion of $z = 1 - x^2 - y^2$ above the plane $z = 0$. Find a vector field $\vec{F} = \vec{F}(x, y, z)$ which is not identically zero such that the flux integral

$$\int \int_S \vec{F} \cdot \vec{n} dS$$

is equal to zero.

16. Consider the velocity field of a fluid $\vec{F} = x\vec{i}$, and use Gauss' divergence theorem in the plane to calculate the amount of fluid which leaves the box with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$ in the unit time.

17. Consider the line integral

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the closed curve intersection of the sphere with radius 1 with the plane $y = x$ oriented so that from the y axis one sees it counter-clockwise. Let $\vec{F}$ be $\vec{F} = y\vec{i}$. Write a surface integral which has the same value as this line integral. (Do not calculate it)