MATH 265 Section E1 Practice Test Number 4

Problem 1 (25 points)

Consider the helix described by parametric equations

$$x = 3cos(t), \qquad y = 3sin(t), z = 2t, \qquad 0 \leq t \leq 40,$$

and assume it has a density given by $\delta = \frac{1}{1+z^2}$. Calculate its mass.

Problem 2 (25 points)

Consider the vector field

$$\vec{F} = 2xy\vec{i} + (x^2 + z)\vec{j} + (y + 2z)\vec{k}.$$

a) Verify that this Vector field is conservative and find the potential function.

b) Calculate

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is the curve $y = x^2$, $z \equiv 0$ in the $x - y$ plane, with $0 \leq x \leq 2$, i.e. with initial point at $x = 0$ and final point at $x = 2$.

Problem 3 (25 points) Consider the surface given by the cylinder

$$z^2 + x^2 = 1, \qquad 0 \leq y \leq 2,$$

with density in $Mass/Surface$, $\delta = yz^2$. Calculate its mass.

Problem 4 (25 points)

Use Gauss divergence theorem to calculate the flux of the vector field $\vec{F} = 2x^2\vec{i} + xy\vec{j} + zx\vec{k}$ across the boundary of the tetrahedron with vertices in the points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$.