

## MATH 317 Section A, Practice Test Number 4

### Problem 1. (25 points)

Consider the following transformations between vector spaces. Say which one of them is a *linear* transformation and which one is not. Also say which one is a *linear operator* and which one is not.

For the ones that are linear transformations say which ones are *onto*, which ones are one-to-one and which ones are isomorphisms.

Justify your answers by citing a definition, a theorem and-or by giving a counterexample.

1. The map

$$\mathcal{L} : M_{n,n} \rightarrow M_{n,n},$$

given by

$$\mathcal{L}(A) = A^2.$$

2. The map

$$\mathcal{L} : \mathbf{R}^n \rightarrow \mathbf{R}^n,$$

defined by

$$\mathcal{L}(\vec{v}) = 2\vec{v}.$$

3. The map

$$\mathcal{L} := \mathcal{P}_2 \rightarrow \mathcal{P}_2$$

defined by

$$\mathcal{L}(p) = p''$$

4. The map

$$\mathcal{L} := \mathbf{R} \rightarrow \mathbf{R}$$

defined by

$$\mathcal{L}(x) = x^2.$$

### Problem 2 (25 points)

Consider the linear operator

$$\mathcal{L} := \mathcal{P}_2 \rightarrow \mathcal{P}_2$$

defined by

$$\mathcal{L}(p) = p' + 2p,$$

and the standard basis  $S := \{1, x, x^2\}$  in  $\mathcal{P}_2$ .

a) Calculate the matrix  $A_S$  representing the linear transformation in this basis.

b) Use the above calculated matrix and coordinatization in  $\mathcal{P}_2$  to calculate

$$\mathcal{L}(x^2 + 1)$$

and

$$\mathcal{L}(x + 2 - x^2)$$

c) Consider the alternative basis  $B = \{1 + x, x, -x^2\}$  and calculate the matrix of the linear transformation with respect to this basis  $A_B$ .

**Problem 3** (25 points)

Let  $\mathcal{U}$  be the vector space of  $2 \times 2$  upper triangular matrices. Consider the matrix

$$A = \begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix},$$

and the linear operator

$$\mathcal{L}(F) = AF,$$

with  $F$  in the space  $\mathcal{U}$ .

a) Use coordinatization to calculate *Range* and *Kernel* of this linear transformation.

b) Show that the dimension theorem is verified.

c) Calculate eigenvalues and eigenvectors of this linear transformation.

d) Is the given transformation diagonalizable? Why?

**Problem 4** (25 points)

Let  $L_1 : \mathcal{V} \rightarrow \mathcal{W}$  and  $L_2 : \mathcal{W} \rightarrow \mathcal{X}$  be two linear transformations.

a) Show that

$$\text{Ker}(L_1) \subseteq \text{Ker}(L_2 \circ L_1)$$

b) Show that

$$\text{Range}(L_2 \circ L_1) \subseteq \text{Range}(L_2)$$