Problem 1 (35 points)

a) Calculate a fundamental set of solutions of the system of differential equations
\[ \vec{x}' = A\vec{x}, \]
where \( A \) is the following matrix
\[
A = \begin{pmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 1 & 2 \\
\end{pmatrix}.
\]

b) Calculate the particular solution corresponding to the initial condition
\[ \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \]

Problem 2 (25 points)

Calculate \( e^A \) where \( A \) is the matrix
\[
A = \begin{pmatrix}
1 & 2 \\
-2 & 1 \\
\end{pmatrix}.
\]

Problem 3 (40 points)

Consider the inhomogeneous system of equations
\[ \vec{x}' = A\vec{x} + \vec{f}, \]
with
\[
A = \begin{pmatrix}
1 & 1 \\
0 & 1 \\
\end{pmatrix},
\]
and
\[
\vec{f} = \begin{pmatrix} e^t \\ 0 \end{pmatrix}.
\]

Calculate the solution corresponding to initial condition
\[ \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \]