MATH 267 Sections A3-C1 Practice Test Number 4

Problem 1 (35 points)

a) Calculate a fundamental set of solutions of the system of differential equations

$$\vec{x}' = A\vec{x},$$

where A is the following matrix

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}.$$

b) Calculate the particular solution corresponding to the initial condition

$$\vec{x}(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 2 (25 points)

Calculate e^A where A is the matrix

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}.$$

Problem 3 (40 points)

Consider the inhomogeneous system of equations

$$\vec{x}' = A\vec{x} + \vec{f},$$

with

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

and

$$\vec{f} = \begin{pmatrix} e^t \\ 0 \end{pmatrix}.$$

Calculate the solution corresponding to initial condition

$$\vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$