

MATH 267 Section E1 Practice Test Number 3

Problem 1(20 points) Calculate the following improper integral

$$\int_0^{\infty} \frac{t}{t^4 + 1} dt.$$

Problem 2(20 points) Use the method of the Laplace transform to solve the following boundary value problem

$$y''' - y = t^3, \quad y(0) = y'(0) = y''(0) = 0.$$

Problem 3(20 points) Calculate the Laplace transform of the function $h(t)$ defined by

$$\begin{aligned} h(t) &= t, & 0 < t \leq 1, \\ h(t) &= t^2, & 1 < t. \end{aligned}$$

Problem 4(20 points) Solve the following boundary value problem

$$y' - y = h(t) - \delta(t - 4), \quad y(0) = 0,$$

where $h(t)$ is the function

$$h(t) = 1, \quad 0 \leq t < 2, \quad h(t) \equiv 0 \quad t \notin [0, 2).$$

Problem 5(20 points) a) Calculate the function

$$h(t) = t \star t^2,$$

where \star is the convolution integral.

- b) Verify that $t \star t^2 = t^2 \star t$.
- c) Calculate the Laplace transform of $h(t)$