MATH 317 Section A, Practice Test Number 3

Problem 1.(20 points)

Consider the following sets with operations $+$, vector sum, and $\cdot$, scalar multiplication. Say which one is a vector space and which one is not. In case the set is **not** a vector space find at least one of the vector space axioms which is not verified. In the case **it is** a vector space show that it is a vector space by showing that

1) It is a nonempty subset of a known vector space.

2) The closure properties of the given operations hold.

1. The $m \times n$ matrix 0, where $+$ is given by matrix addition and $\cdot$ is given by multiplication of a matrix by a scalar.
2. The set of functions defined on $\mathbf{R}$ such that $f(0) = 0$ where $+$ is the usual sum of functions, i.e. $(f+g)(x) = f(x)+g(x)$, and the scalar multiplication is the usual product of a scalar by a function i.e. $(af)(x) = a(f(x))$, for a function f and scalar a.
3. The set of functions defined on $\mathbf{R}$ such that $f(0) = 1$ where $+$ is the usual sum of functions, i.e. $(f+g)(x) = f(x)+g(x)$, and the scalar multiplication is the usual product of a scalar by a function i.e. $(af)(x) = a(f(x))$, for a function f and scalar a.
4. The set of 2×2 matrices with determinant equal to zero where $+$ is given by matrix addition and $\cdot$ is given by multiplication of a matrix by a scalar.
5. The set of vectors $\vec{v}$ in $\mathbf{R}^4$ such that the dot product with $\vec{x} = [9, 1, 2, 1]$ is zero, with the usual vector sum and multiplication by a scalar.

Problem 2 (20 points)

Consider the subset of $\mathcal{P}_2$

$$S = \{1 + x^2, 2 - x + x^2, 1 - x\}.$$

Find a basis of $span(S)$ which contains only elements in S. What is the dimension of $span(S)$? (Hint: use the expression of the elements in S with respect to the standard basis in $\mathcal{P}_2$).

Problem 3 (20 points)

Check which ones of these sets of vectors are linearly independent and which ones are not in their vector spaces

1. The vectors $\vec{v}_1 = [0, 1, 2]$, $v_2 = [1, -1, 1]$, $v_3 = [2, 1, 1]$ in $\mathbf{R}^3$.
2. The matrices $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 11 & 11 \\ 1 & 0 \end{pmatrix}$ in $\mathcal{M}_{2,2}$.

Problem 4 (20 points)
Consider $\mathbf{R}^3$ and two ordered bases

$$O = \{[1,2,1],[1,0,0],[0,1,1]\},$$

$$N = \{[1,1,0],[1,2,0],[0,0,1]\},$$

Find the transition matrix P_{ON} which transforms coordinates with respect to O to coordinates with respect to N, i.e. for every vector $\vec{v}$ in $\mathbf{R}^3$

$$[\vec{v}]_N = P_{ON}[\vec{v}]_O.$$

Problem 5 (20 points) Consider a finite dimensional vector space $\mathcal{V}$ with an ordered basis B. Prove that r, vectors $\vec{v}_1, ..., \vec{v}_r$ are linearly independent in $\mathcal{V}$ if and only if $[\vec{v}_1]_B, ..., [\vec{v}_r]_B$ are linearly independent in $\mathbf{R}^n$.