MATH 267 Sections A3-C1 Practice Test Number 2

Problem 1.(25 points) Consider the following initial value problem (IVP)

$$y'' + y - x = 0, \qquad y(0) = 1, y'(0) = 0.$$

Find the solution of this IVP as a power series

$$y = \sum_{n=0}^{\infty} a_n x^n,$$

by

1. Finding a recursive formula for the coefficients a_n.
2. Finding an explicit formula for the coefficients a_n.

Problem 2.(25 points)

Use the method of the Laplace transform to solve the following initial value problem

$$y'' - 4y' + 4y = t + 1, \qquad y(0) = 0, y'(0) = 1.$$

Problem 3.(25 points)

Calculate the Laplace transform of the following discontinuous function by first expressing it in terms of the Heaviside function.

$$f(t) = e^t, \qquad 0 < t \leq 1,$$

$$f(t) = t, \qquad 1 < t \leq 2,$$

$$f(t) = 1, \qquad 2 < t < \infty.$$

Problem 4. (25 points) Use the Laplace transform to solve the following initial value problem. Notice the initial point is not zero and $H_{3,4}$ is an interval function.

$$y'' + y = H_{3,4}, \qquad y(2) = y'(2) = 0$$