MATH 317 Section A, Practice Test Number 2

Problem 1.(20 points)

Assume A is an $n \times n$ **nonsingular** matrix. Which of the following statements **must** be false, which one **may** be true and which one **must** be true. Justify your answer and in the case where it may be true give an example.

1. A is an upper triangular matrix.
2. The reduced row echelon form of A is the identity.
3. The standard vector
$$e_1 := \begin{pmatrix} 1 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}$$
is a solution of the system
$$AX = 0$$
4. $$det A = 0$$

Problem 2(20 points) Consider the matrix

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

a) Calculate the determinant and conclude that A has an inverse.

b) Use Gauss-Jordan elimination to find A^{-1}

c) Use the inverse to calculate the solution of the system

$$AX = B$$

where $B := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Problem 3(20 points) Calculate the determinant of the following matrix by using expansion along the second column

$$A := \begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}.$$

Problem 4(20 points)

a) Diagonalize the following 2×2 matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

b) Calculate A^6 using the diagonalization of A

c) Show that the following 3×3 matrix is not diagonalizable.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Problem 5(20 points) Consider a diagonal matrix A, $n \times n$.

a) Prove that A is nonsingular if and only if all the elements on the diagonal are different from zero.

b) Prove that $A^{-1} = B$ where B is also diagonal and $b_{ii} = \frac{1}{a_{ii}}$, $i = 1, ..., n$.