

MATH 267 Section E1 Practice Test Number 1

Problem 1.(20 points) Solve the following linear initial value problem using the method of variation of parameters.

$$\begin{aligned}x' &= \cos(t)x + \cos(t), \\x(0) &= 3.\end{aligned}$$

Problem 2.(20 points) Find the general solution of the following differential equation

$$\frac{dy}{dx} = \frac{1}{x^2y}.$$

Problem 3 (20 points) Apply the existence and uniqueness theorem to give an interval of existence for the solution of the following boundary value problem.

$$\begin{aligned}y' &= \frac{1}{t^2 - 3t + 2}y + \cos(t), \\y(1.5) &= 1\end{aligned}$$

Problem 4 (20 points) Consider the logistic model of population dynamics

$$y' = y(3 - 100y).$$

Find the unstable and stable equilibrium points. Discuss the asymptotic behavior of the solutions for various initial values of the population $y(0) \geq 0$.

Problem 5 (20 points) Verify that the following differential equation is not exact. Find an integrating factor and give the general solution of the equation.

$$y' = e^{2x} + y - 1.$$