MATH 267 Section E1 Practice Test Number 1

Problem 1. (20 points) Solve the following linear initial value problem using the method of variation of parameters.

\[ x' = \cos(t)x + \cos(t), \]
\[ x(0) = 3. \]

Problem 2. (20 points) Find the general solution of the following differential equation

\[ \frac{dy}{dx} = \frac{1}{x^2y}. \]

Problem 3 (20 points) Apply the existence and uniqueness theorem to give an interval of existence for the solution of the following boundary value problem.

\[ y' = \frac{1}{t^2 - 3t + 2}y + \cos(t), \]
\[ y(1.5) = 1 \]

Problem 4 (20 points) Consider the logistic model of population dynamics

\[ y' = y(3 - 100y). \]

Find the unstable and stable equilibrium points. Discuss the asymptotic behavior of the solutions for various initial values of the population \( y(0) \geq 0 \).

Problem 5 (20 points) Verify that the following differential equation is not exact. Find an integrating factor and give the general solution of the equation.

\[ y' = e^{2x} + y - 1. \]