MATH 265 Section E1, Test Number 1

Problem 1(25 points)
Consider the parametric curve in the plane

$$x = sin^2(t)$$

$$y = cos^2(2t)$$

a) Eliminate t to express y as a function of x.

b) Argue that in the interval $0 < t < \frac{\pi}{2}$ this gives a well defined function $y = y(x)$ and calculate $\frac{dy}{dx}$ as a function of t.

Problem 2 (25 Points) Let the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ be defined as follows

$$\vec{u} = \vec{i} + 3\vec{j},$$

$$\vec{v} = -\vec{i} + 2\vec{j},$$

$$\vec{w} = -\vec{i} + \vec{k}.$$

Calculate the following expressions

a)

$$(2\vec{u} + 5\vec{w}) \times \vec{v},$$

b)

$$(2(\vec{u} \times \vec{v}) \cdot \vec{w})\vec{w},$$

c)

$$(\vec{u} \cdot \vec{v})\vec{w} \times \vec{u}$$

Problem 3 (25 Points)[1] Consider the parametric curve in space given by

$$\vec{r}(t) = t^2\vec{i} + \frac{4}{3}t^{\frac{3}{2}}\vec{j} + t\vec{k},$$

in the interval $0 \leq t \leq 2$.

a) Calculate the arclentgh of this curve $s(t)$ as a function of t. What is the length of the curve?

b) Calculate the curvature $k(t)$ as a function of t.

Problem 4 (25 Points) Give the equation of the plane parallel to the two vectors

$$\vec{u} = \vec{i} + \vec{j},$$

and

$$\vec{v} = \vec{k},$$

and containing the point $P = (1, 2, 1)$.

[1]Please correct the final solution to part b) in the solutions. The correct solution is $\frac{1}{(2t+1)^3}\sqrt{\frac{1+4t^2 4t^3}{t}}$