MATH 265 Section E1, Test Number 1

Problem 1 (25 points)
Consider the parametric curve in the plane
\[ x = \sin^2(t) \]
\[ y = \cos^2(2t) \]

a) Eliminate \( t \) to express \( y \) as a function of \( x \).
b) Argue that in the interval \( 0 < t < \frac{\pi}{2} \) this gives a well defined function \( y = y(x) \) and calculate \( \frac{dy}{dx} \) as a function of \( t \).

Problem 2 (25 Points) Let the vectors \( \vec{u}, \vec{v} \) and \( \vec{w} \) be defined as follows
\[ \vec{u} = \vec{i} + 3\vec{j}, \]
\[ \vec{v} = -\vec{i} + 2\vec{j}, \]
\[ \vec{w} = -\vec{i} + \vec{k}. \]

Calculate the following expressions
a) \( (2\vec{u} + 5\vec{w}) \times \vec{v}, \)
b) \( 2(\vec{u} \times \vec{v}) \cdot \vec{w}; \)
c) \( (\vec{u} \cdot \vec{v})\vec{w} \times \vec{u}. \)

Problem 3 (25 Points) Consider the parametric curve in space given by
\[ \vec{r}(t) = t^2\vec{i} + \frac{2\sqrt{2}}{3}t^3\vec{j} + t\vec{k}, \]
in the interval \( 0 \leq t \leq 2. \)

a) Calculate the arclength of this curve \( s(t) \) as a function of \( t \). What is the length of the curve?
b) Calculate the curvature \( k(t) \) as a function of \( t \).

Problem 4 (25 Points) Give the equation of the plane parallel to the two vectors
\[ \vec{u} = \vec{i} + \vec{j}, \]
and
\[ \vec{v} = \vec{k}, \]
and containing the point \( P = (1, 2, 1) \).