

MATH 265 Section E1, Test Number 1

Problem 1(25 points)

Consider the parametric curve in the plane

$$x = \sin^2(t)$$

$$y = \cos^2(2t)$$

- a) Eliminate t to express y as a function of x .
b) Argue that in the interval $0 < t < \frac{\pi}{2}$ this gives a well defined function $y = y(x)$ and calculate $\frac{dy}{dx}$ as a function of t .

Problem 2 (25 Points) Let the vectors \vec{u} , \vec{v} and \vec{w} be defined as follows

$$\vec{u} = \vec{i} + 3\vec{j},$$

$$\vec{v} = -\vec{i} + 2\vec{j},$$

$$\vec{w} = -\vec{i} + \vec{k}.$$

Calculate the following expressions

a)

$$(2\vec{u} + 5\vec{w}) \times \vec{v},$$

b)

$$(2(\vec{u} \times \vec{v}) \cdot \vec{w})\vec{w},$$

c)

$$(\vec{u} \cdot \vec{v})\vec{w} \times \vec{u}$$

Problem 3 (25 Points) Consider the parametric curve in space given by

$$\vec{r}(t) = t^2\vec{i} + \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\vec{j} + t\vec{k},$$

in the interval $0 \leq t \leq 2$.

a) Calculate the arclength of this curve $s(t)$ as a function of t . What is the length of the curve?

b) Calculate the curvature $k(t)$ as a function of t .

Problem 4 (25 Points) Give the equation of the plane parallel to the two vectors

$$\vec{u} = \vec{i} + \vec{j},$$

and

$$\vec{v} = \vec{k},$$

and containing the point $P = (1, 2, 1)$.