

MATH 267 Sections A3-C1 Practice Test Number 1

Problem 1.(25 points) Solve the following linear initial value problem using the method of variation of parameters.

$$\begin{aligned}x' &= \cos(t)x + \cos(t), \\x(0) &= 3.\end{aligned}$$

Problem 2.(25 points) Find the general solution of the following differential equation by applying the method for homogeneous equations

$$(x + y)dy + (y - x)dx = 0.$$

Problem 3 (25 points) The following differential equation is not exact. Find an integrating factor and give the general solution of the equation.

$$y' = e^{2x} + y - 1.$$

Problem 4 (25 points)

Consider the following autonomous differential equation

$$x' = tg^{-1}(x - 1)$$

- Using the theorems of existence and uniqueness argue that the solution corresponding to the initial condition $x(0) = 0$ exists and is unique.
- Draw the phase line and classify the stable and unstable equilibrium point(s).
- Consider an equilibrium solution $x_e(t) \equiv x_0$ and the solution, $x(t)$ corresponding to $x(0) = 0$. Give a bound for $|x(t) - x_0|$ for $t = 1$.