Math 265 Practice Test # 4 Solutions.

Pr 1

$$M = \int_C \frac{1}{1+z^2}\, ds = \int_0^{40} \frac{1}{1+(2t)^2} \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (2)^2}\, dt$$

$$= \frac{\sqrt{13}}{2}\Big[\arctan(2t)\Big]_0^{40} = \frac{\sqrt{13}}{2}\arctan(80)$$

Pr 2

a) $M = 2xy \quad N = (x^2+z) \quad P = (y+2z)$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \qquad 2x = 2x \ \checkmark$$

$$\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \qquad 0 = 0 \ \checkmark$$

$$\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} \qquad 1 = 1 \ \checkmark$$

$$f_x = 2xy$$

$$f_y = x^2 + z$$

$$f_z = y + 2z$$

$$f = \int 2xy\,dx + C(y,z) =$$

$$= x^2 y + C(y z)$$

$$f_y = x^2 + z = x^2 + \frac{\partial C}{\partial y} \Rightarrow C(y z) = \int z\,dy + C_1(z$$

$$C(y,z) = zy + C_1(z)$$

$$f = x^2 y + zy + C_1(z)$$

$$f_z = y + \frac{\partial C_1}{\partial z} = y + 2z \Rightarrow C_1 = \int 2z\,dz + C$$

$\Rightarrow C_1(z) = z^2 + C$

can set to zero

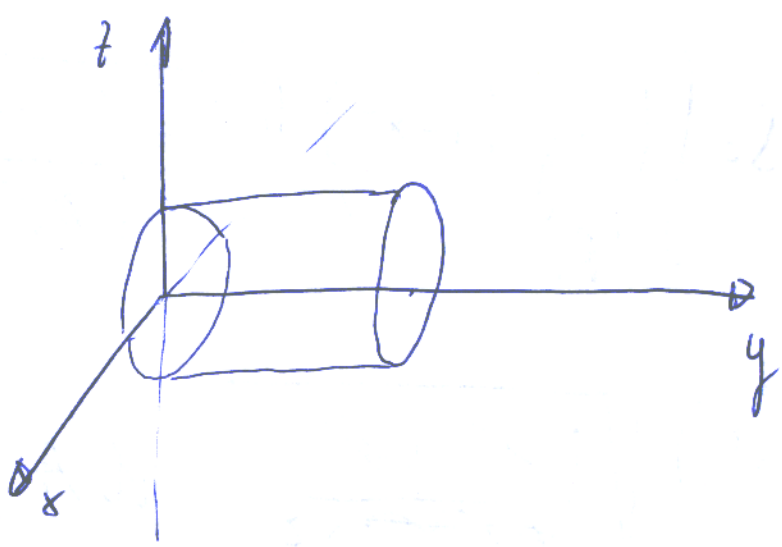
$f = x^2 y + zy + z^2 + \cancel{C}$

b) when $x = 2$ $y = 4$

$f(2,4,0) - f(0,0,0) =$

$[(4 \cdot 4) + (0) + (0)] - [0] = \boxed{16}$

Pr 3



Separate the cylinder into two parts $z \geq 0$

and $z<0$, by symmetry of density mass of upper part and mass of lower part are the same so.

$$m = 2 \iint_S y z^2 \, dS$$

$$S = \{(x, y, z) \mid (x, y) \text{ in } R, \ z = \sqrt{1-x^2} \}.$$

$$R = \{(x, y) \mid -1 \le x \le 1, \ 0 \le y \le 2\}$$

$$\iint_S y z^2 \, dS = \iint_R y(1-x^2) \sqrt{1 + \left(\frac{1(-2x)}{2\sqrt{1-x^2}}\right)^2 + (0)^2} \, d$$

$$\iint_R y(1-x^2) \sqrt{\frac{1 \cancel{-x^2} + \cancel{x^2}}{(\sqrt{1-x^2})^2}} \, dA$$

$$\iint_R y\sqrt{1-x^2}\,dA =$$

$$\int_0^2 y \int_{-1}^{1} \sqrt{1-x^2}\,dx\,dy$$

set $x = \cos t$ $\quad$ $x=-1 \Longleftrightarrow t=\pi$ $\quad$ $x=1 \Longleftrightarrow t=$

$dx = -\sin t\,dt$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = -\int_{\pi}^{0} \sin^2 t\,dt = \int_0^{\pi} \sin^2 t\,dt =$$

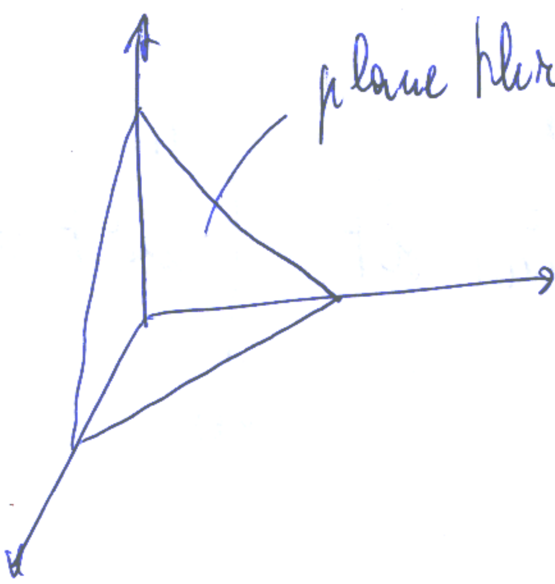
$$\int_0^{\pi} \frac{1-\cos(2t)}{2}\,dt = \frac{\pi}{2}$$

$$\frac{\pi}{2}\int_0^2 y\,dy = \frac{\pi}{4}\left[y^2\right]_0^2 = \pi$$

$$\boxed{\text{mass} = 2 \times \pi}$$

Pr 4 $\quad \operatorname{div}\vec{F} = 4x + x + x = 6x$

$$\iiint_V \operatorname{div}\vec{F} = \iiint_V 6x\, dV$$



plane through points $(1,0,0)$
$(0,1,0)$
$(0,0,1)$

plane $z = ax + by + d$

$\left.\begin{matrix} x=0 \\ y=0 \\ z=1 \end{matrix}\right\} \Rightarrow d = 1 \qquad \left.\begin{matrix} x=0 \\ z=0 \\ y=1 \end{matrix}\right\} \Rightarrow b = -1 \qquad \left.\begin{matrix} y=0 \\ z=0 \\ x=1 \end{matrix}\right\} \Rightarrow a = -$

Equation for plane $z = 1 - x - y$

$V = \{(x,y,z) \mid (x,y) \text{ in } D,\ 0 \leq z \leq 1-x-y\}$

$D = \{(x,y) \mid 0 \leq x \leq 1\ 0 \leq y \leq 1-x\}$.

$$\iiint_V 6x \, dV = \iint_D \int_0^{1-x-y} 6x \, dz \, dA =$$

$$\iint_D 6x(1-x-y) \, dA$$

$$= \int_0^1 \int_0^{1-x} 6x(1-x) - 6xy \, dy \, dx =$$

$$\int_0^1 6x(1-x)^2 - 6\frac{x(1-x)^2}{2} \, dx$$

$$3\int_0^1 x(1-x)^2 \, dx = 3\int_0^1 x - 2x^2 + x^3 \, dx = 3\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) =$$

$$= \frac{3}{12} = \boxed{\frac{1}{4}}$$