MATH 267 (Section E1) Homework No. 2

Reading

Section 2.4,
Section 2.5 (up to Example 1 included)
Section 2.6
Section 2.7

Suggested Problems

Section 2.4: Exercise 1,2,3,7,9
Section 2.6: Exercises 1, 4, 11, 13, 19
Section 2.7: Exercise 1,3

Problems to be handed in in class on Thursday January 25

Problem 1 Consider the boundary value problem

$$y' = \frac{t}{\sqrt{1-ty}}$$
$$y(0) = 1.$$

Find a box in the (t, y)-plane containing a curve which is solution of the above B.V.P. Justify your answer.

Problem 2 Show that the following differential equation is exact

$$xy^2 + sin(y) + (x^2y + cos(y)x)y' = 0.$$

Find in implicit form the function satisfying this differential equation and the boundary condition

$$y(1) = \pi.$$

Problem 3 Consider the B.V.P

$$y' = y^2 + t,$$
$$y(0) = 1.$$

Apply Euler method with step $h = 0.1$ to find approximations of $y(0.1)$, $y(0.2)$ and $y(0.3)$.

Pr 3 iteration $y_n = y_{n-1} + f(t_{n-1}, y_{n-1})h$. $t_0 = 0$ $y_0 =$

$h = 0.1$

$f = y^2 + t$

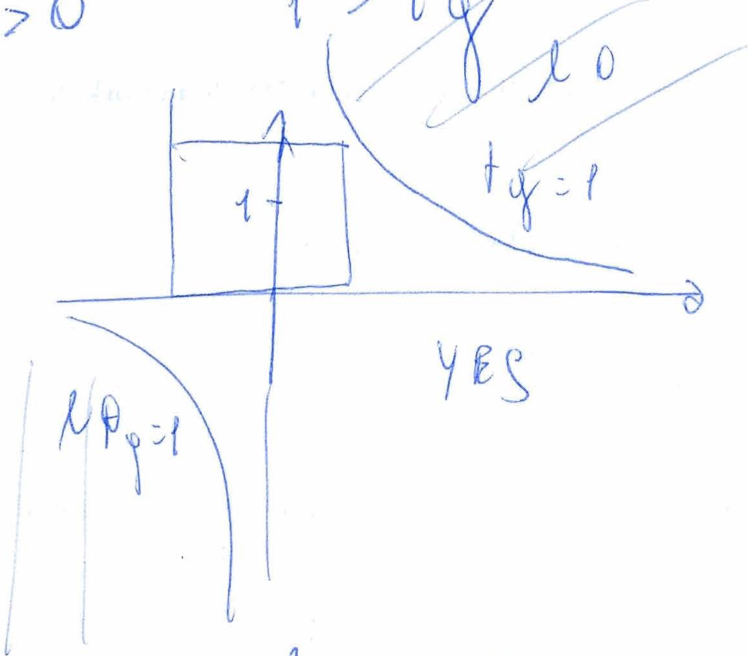
$y_1 = 1 + (1^2 + 0)0.1 = 1.1$

$y_2 = 1.1 + ((1.1)^2 + 0.1)0.1 = 1.231$

$y_3 = 1.231 + ((1.231)^2 + 0.2)0.1 \simeq 1.4025$

HW Solutions

① Need $1-ty>0$ $\quad 1>ty$



YES

② $M = xy^2 + \sin(y)$ $\quad N = (x^2y + \cos(y)x)$

$M_y = 2xy + \cos(y)$ $\quad N_x = 2xy + \cos(y)$

$M_y = N_x$ ✓ $\Rightarrow$ exact.

$\psi = \int xy^2 + \sin(y)\,dx + h(y) = \frac{1}{2}x^2y^2 + \sin y\, x + h(y)$

$\frac{\partial \psi}{\partial y} = \cancel{x^2y} + \cancel{\cos y\, x} + h'(y) = \cancel{x^2y} + \cancel{\cos(y)\, x} \Rightarrow h(y) =$

solution

$\frac{1}{2}x^2y^2 + \sin(y)x = C$ $\quad x=1 \quad y=\pi \Rightarrow C=\frac{\pi^2}{2}$

$$\boxed{\frac{1}{2}x^2y^2 + \sin(y)x = \frac{\pi^2}{2}}$$