HWK # 9 Solutions

1

Problem 5.2 #4

$$A = \begin{bmatrix} 5 & -3 \\ -4 & 3 \end{bmatrix} \qquad A - \lambda I = \begin{bmatrix} 5-\lambda & -3 \\ -4 & 3-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (5-\lambda)(3-\lambda) - 12 =$$

$$15 - 5\lambda - 3\lambda + \lambda^2 - 12 = \lambda^2 - 8\lambda + 3$$

Characteristic polynomial $= \lambda^2 - 8\lambda + 3$

$$\lambda_{1/2} = 4 \pm \sqrt{16 - 3}$$

eigenvalues $\lambda_1 = 4 + \sqrt{13}$

$\lambda_2 = 4 - \sqrt{13}$

Problem 5.2 # 18

$$A - 5I = \begin{bmatrix} 0 & -2 & 6 & -1 \\ 0 & -2 & h & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix} \sim$$

$$\begin{bmatrix} 0 & -2 & 6 & -1 \\ 0 & 0 & h-6 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

Need 2 Pivot columns and 2 NonPivot columns

$\boxed{h = 6}$ otherwise 3 Pivots

Problem 5.3 # 12

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix} \qquad \lambda_1 = 2 \qquad \lambda_2 = 8$$

$$\lambda_1 = 2 \quad \begin{bmatrix} 4-2 & 2 & 2 \\ 2 & 4-2 & 2 \\ 2 & 2 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$(A - 2I)\vec{v} = \vec{0} \Rightarrow \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Geometric multiplicity = 2

$A - 8I =$

$$\begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix} \sim \begin{bmatrix} -4 & 2 & 2 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix} \sim$$

$$\begin{array}{c} \begin{array}{ccc} P & P & NP \end{array} \\ \sim \begin{bmatrix} -4 & 2 & 2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = 1 \qquad x_2 = 1$

$-4x_1 + 2x_2 + 2x_3 = 0 \Rightarrow x_1 = 1$

3

eigenvector $\vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Problem 5.3 26

Yes it is possible. The third eigenvalue must have geom. mult plicity $= 2$. If it has geometric multiplicity $= 1$ the ~~matrix~~ matrix is not diagonalizable $\measuredangle$