

MATH 307 Section C, Homework No. 8

Reading

Section 4.4, 4.7 and notes on line on Kernel and Range of a Linear Transformation.

Suggested Exercises

Section 4.4: Exercise 1, 3, 13, 31.

Additional Problem: Consider the vector space of 2×2 upper triangular matrices and the basis

$$B = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}.$$

Find the coordinates for the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}.$$

$$\text{(Answer: } [A]_B = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \text{).}$$

Section 4.7: Exercise 1, 5, 7, 9, 13.

More additional problems on Kernel and Range of a Linear transformation

Problem 1 Consider the linear transformation $T : P_2 \rightarrow P_2$, $T(p) := t \frac{d}{dt} p$. Find a basis for Kernel and Range of this linear transformation.

(Answer: $Ker = span\{1\}$, $Range = span\{t, t^2\}$).

Problem 2 Consider the linear transformation $T : M_{2,2} \rightarrow M_{2,2}$,

$$T(p) := 2(A - A^T)$$

. Find a basis for Kernel and Range of this linear transformation.

Answer:

$$Ker = span \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}, \quad Range = span \left\{ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\}$$