HWK #7 Solutions

Problem 4.3 #4

Set is a basis iff it is linearly independent. Check l.i. by row reduction method.

$$\begin{bmatrix} 2 & 1 & -7 \\ -2 & -3 & 5 \\ 1 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & -7 \\ 0 & -2 & -2 \\ 0 & \frac{3}{2} & \frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & -7 \\ 0 & -2 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

Yes l.i. It is a basis.

Problem 4.3 #8

Not a basis because not lin. ind. But set spans $\mathbb{R}^3$ because $\begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$ are lin independent.

check by calculating.

$$\det \begin{bmatrix} 1 & 0 & 0 \\ -4 & 3 & 2 \\ 3 & -1 & -2 \end{bmatrix} \neq 0 .$$

Problem 4.3 #20 The space has dimension $< 3$ because $\vec{v}_3$ is a linear combination of $\vec{v}_1$ and $\vec{v}_2$. However $\{\vec{v}_1, \vec{v}_2\}$ is a basis because 1) they are lin. ind.

$$\begin{bmatrix} 7 & 4 \\ 4 & -7 \\ -8 & 2 \\ -5 & 5 \end{bmatrix} \rightarrow \det \neq 0$$

2) Every element is a linear combination of $\vec{v}_1, \vec{v}_2$. In fact

$$\vec{v} = \underbrace{l.c.}_{\text{linear combination}}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$$

but $\vec{v}_3 = l.c.(\vec{v}_1$

So $\vec{v}_9 = l.c.(\vec{v}_1, \vec{v}_2, l.c.(\vec{v}_1, \vec{v}_2)) = l.c.(\vec{v}_1, \vec{v}_2$

Problem 4.3 # 34

$(1+t) + (1-t) = 2$ so not lin. ind.

Every vector in the span is

$$a(1+t) + b(1-t) + c\,2 =$$

$$(a+b+2c) + (a-b)t$$

basis $\{1, t\}$.

Problem 4.4 #14

$$3+t-6t^2 = a(1-t^2)+b(t-t^2)+c(2-2t+t^2)$$

$$\left.\begin{aligned} 3 &= a+2c \\ 1 &= b-2c \\ -6 &= -a-b+c \end{aligned}\right\}$$

Solve system

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 1 \\ -1 & -1 & 1 & -6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$c=-2$ $\quad b=-3$ $\quad a=7$

$$\begin{pmatrix} 7 \\ -3 \\ -2 \end{pmatrix}$$

Problem 4.6 #4

Pivot columns 1, 2, 4

Non Pivot columns 3, 5, 6

rank $A = 3$

dim Nul $A = 3$

Basis for col(A) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \\ -3 \\ -2 \end{pmatrix}, \begin{pmatrix} 7 \\ 10 \\ 1 \\ -5 \\ 0 \end{pmatrix} \right\}$

To find basis for Nul A solve

$B\vec{x} = \vec{0}$ $\quad x_3 = \lambda \quad x_5 = \mu \quad x_6 = \nu$.

$$x_1 + x_2 - 3\lambda + 7 x_4 + 8\mu - 8\nu = 0$$

$$x_2 - \lambda + 3x_4 + 4\mu - 3\nu = 0$$

$$x_4 - \mu - 2\nu = 0$$

$$\boxed{x_4 = \mu + 2\nu.}$$

$$x_2 - \lambda + 3(\mu + 2\nu) + 4\mu - 3\nu = 0$$

$$x_2 - \lambda + 3\mu + 6\nu + 4\mu - 3\nu = 0$$

$$\boxed{x_2 = \lambda - 7\mu - 3\nu}$$

$$x_1 + \lambda - 7\mu - 3\nu - 3\lambda + 7(\mu + 2\nu) + 9\mu - 9\nu = 0$$

$$x_1 - 2\lambda + 9\mu + 2\nu = 0$$

$$\boxed{x_1 = 2\lambda - 9\mu - 2\nu}$$

$$\begin{array}{ccccc} \lambda & & \mu & & \nu \\ \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} & , & \begin{pmatrix} -9 \\ -7 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} & , & \begin{pmatrix} -2 \\ -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

Basis for the row space of A.

$$\left\{ \begin{pmatrix} 1 & 1 & -3 & 7 & 8 & -9 \end{pmatrix}, \begin{pmatrix} 0 & 1 & -1 & 3 & 4 & -3 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 & -1 & -2 \end{pmatrix}. \right\}$$