

## MATH 267 (Section E1) Homework No. 7

### Reading

Sections 6.5, 6.6 (omit proof of Theorem 6.6.1), 7.2, 7.3.

### Suggested Problems

Section 6.5: Exercise 1, 2, 5,

Section 6.6: Exercises 1,4,6,8,9,14.

Section 7.2: 10, 21, 22, 25

Section 7.3: 6,8, 12, 15,

### Problems to be handed in in class on Thursday March 29-th

**Problem 1** Solve the boundary value problem

$$y' - y = \cos(t) - \delta(t - 4), \quad y(0) = 0,$$

using the method of the Laplace transform.

**Problem 2** a) Calculate the convolution integral

$$h(t) = e^t \star t = \int_0^t e^{t-\tau} \tau d\tau.$$

b) Calculate the Laplace transform  $H(s)$  of  $h(t)$ .

c) Verify that  $H(s) = \mathcal{L}(e^t)\mathcal{L}(t)$  according to the convolution theorem.

**Problem 3** Consider the matrix function

$$A(t) = \begin{pmatrix} \cos(t) & e^t \\ e^{-t} & \cos(t) \end{pmatrix}.$$

a) Calculate eigenvalues and eigenvectors of  $A(0)$ .

b) Calculate

$$\frac{d}{dt}A(t).$$

c) Calculate

$$\int_0^t A(\tau) d\tau.$$