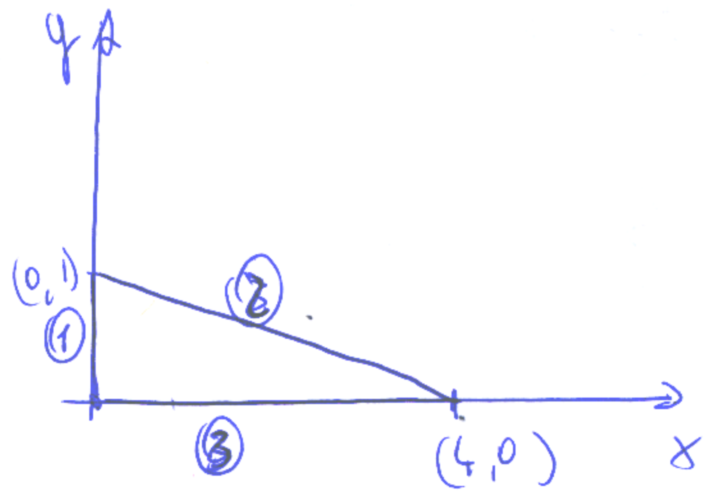
HWK # 6 Solutions

Prob 30 15.8

max and min

$$f = 2x^2 + y^2 - 4x - 2y + 5$$



Find stationary points

$f_x = 4x - 4 = 0 \qquad x = 1$

$f_y = 2y - 2 = 0 \qquad y = 1$

point (1,1) not inside triangle

Find Boundary points of interest

On 1 $\quad x = 0 \quad f = y^2 - 2y + 5$

$0 \le y \le 1$

x	y	f
0	0	5
0	1	4
4	0	21 max
$\frac{32}{33}$	$\frac{25}{33}$	$\frac{68}{33}$ min
1	0	3

$f' = 2y - 2 = 0$

$y = 1$ stationary point is the same by boundary point

On (2) $y = -\frac{1}{4}x + 1$

$f = 2x^2 + \left(1 - \frac{1}{4}x\right)^2 - 4x - 2\left(1 - \frac{1}{4}x\right) + 5 =$

$2x^2 + 1 \cancel{- \frac{1}{2}x} + \frac{1}{16}x^2 - 4x - 2 \cancel{+ \frac{1}{2}x} + 5 =$

$\frac{33}{16}x^2 - 4x + 4 \qquad 0 \le x \le 4$

End point $x=4 \quad y=0$

112

$$f = \frac{33 \cdot \cancel{16}}{\cancel{16}} - 16 + 4 = 21$$

$$f' = \frac{66}{16}x - 4 = 0 \qquad x = \frac{64}{66} = \frac{32}{33}$$

$$y = -\frac{1}{4}\frac{32}{33} + 1 = 1 - \frac{8}{33} = \frac{25}{33}$$

$$f = \frac{\cancel{33}}{16}\left(\frac{32}{33}\right)^2 - 4 \cdot \frac{32}{33} + 4 =$$

$$= \frac{1}{\cancel{16} \cdot 33} 2^2 16^2 - 4 \cdot \frac{32}{33} + 4 =$$

$$-\frac{64}{33} + 4 = \frac{132}{33} - \frac{64}{33} = \frac{68}{33}$$

On ③ $y = 0$

$$f = 2x^2 - 4x + 5$$

$$0 \leq x \leq 4$$

End points already checked.

Stationary point

$$4x - 4 = 0 \Rightarrow x = 1$$

$$f = 2 - 4 + 5 = 3$$

Prob 12 15.9

Rez.

Volum $f = xyz$

st. $g = \frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$$

$$\begin{cases} yz\vec{i} + xz\vec{j} + yx\vec{k} = \lambda\left(\frac{1}{a}\vec{i} + \frac{1}{b}\vec{j} + \frac{1}{c}\vec{k}\right) \\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0 \end{cases}$$

$$\begin{cases} yz = \frac{\lambda}{a} \\ xz = \frac{\lambda}{b} \\ yx = \frac{\lambda}{c} \end{cases} \qquad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0$$

$h = ayz$

$x\cancel{z} = \frac{ay\cancel{z}}{b}$

$\cancel{y}x = \frac{a\cancel{y}z}{c}$

$\frac{\cancel{a}y}{b} = \frac{\cancel{a}z}{c}$

Express everything in terms of y.

$$\frac{\cancel{a}y}{\cancel{a}b} + \frac{y}{b} + \frac{\cancel{c}y}{b\cancel{c}} = 1$$

$$3\frac{y}{b} = 1 \Rightarrow y = \frac{b}{3}$$

$$z = \frac{c}{3}$$

$$x = \frac{a}{3}$$

$$\boxed{\text{Max volume} = \frac{a \cdot bc}{27}}$$

Prob 14 15.8

Max $2r\left(\sin\frac{\alpha}{2}+\sin\frac{\beta}{2}+\sin\frac{\gamma}{2}\right)$

st $\alpha+\beta+\gamma=2\pi$

$\nabla f=\lambda\nabla g$

$\frac{2r}{2}\cos\left(\frac{\alpha}{2}\right)\vec{i}+\frac{2r}{2}\cos\left(\frac{\beta}{2}\right)\vec{j}+\frac{2r}{2}\cos\left(\frac{\gamma}{2}\right)\vec{k}=$

$\lambda\left(\vec{i}+\vec{j}+\vec{k}\right)$

$r\cos\left(\frac{\alpha}{2}\right)=\lambda$

$r\cos\left(\frac{\beta}{2}\right)=\lambda$

$r\cos\left(\frac{\gamma}{2}\right)=\lambda$

$$\begin{cases}\alpha+\beta+\gamma=2\pi & (1)\\ \cos\left(\frac{\alpha}{2}\right)=\cos\left(\frac{\beta}{2}\right)=\cos\left(\frac{\gamma}{2}\right) & (2)\end{cases}$$

As $\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$0 \leq \frac{\alpha}{2}, \frac{\beta}{2}, \frac{\gamma}{2} \leq \pi$

So $\cos\left(\frac{\alpha}{2}\right) = \cos\left(\frac{\beta}{2}\right) = \cos\left(\frac{\gamma}{2}\right) \Rightarrow \alpha = \beta = \gamma$

$3\frac{\alpha}{2} = \pi$ $\boxed{\alpha = \beta = \gamma = \frac{2\pi}{3}}$