HWK No 6 Solutions

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Problem 4.1 # 8

Yes the set of all polynomials in P_n with $p(0) = 0$ is a subspace of P_n because

1) it is not empty since the zero polinomial is there

2) It is closed under sum and multiplication by a scalar since

a) If $p(0) = 0$ and $q(0) = 0$

$(p+q)(0) = p(0) + q(0) = 0$

b) If $p(0) = 0$ and $\lambda \in \mathbb{R}$

$(\lambda p)(0) = \lambda p(0) = \lambda \cdot 0 = 0$

Problem 4.1 # 20

a) The set $C[a,b]$ is non empty since the constant zero function is there. Moreover it is closed under sum and multiplication by a scalar since

- the sum of two continuous functions is continuous
- the product of a continuous function by a scalar is continuous.

b) This set is non empty since the zero function is such that $f(a) = f(b)$

Moreover is closed under + and $\cdot$

In fact assume $f(a) = f(b)$

and $g(a) = g(b)$

then

$$(f+g)(a) = f(a) + g(a) = f(b) + g(b) =$$

$$= (f+g)(b) \quad \checkmark$$

$$\checkmark \quad (\lambda f)(a) = \lambda(f(a)) = \lambda(f(b)) = \lambda f(b$$

Problem 4.2 # 24

$\vec{w}$ is in Col(A) if there exists $\vec{x}$ such that

$A\vec{x} = \vec{w}$ look @ system

$$\begin{bmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix} \sim$$

$\frac{6}{8}(1) + (2) \to (2)$

$\frac{1}{2}(1) + (3) \to (3)$

$$\begin{bmatrix} -8 & -2 & -9 & 2 \\ 0 & \frac{5}{2} & \frac{5}{4} & \frac{5}{2} \\ 0 & -1 & -\frac{1}{2} & -1 \end{bmatrix} \sim$$

$\frac{2}{5}(2) + (3) \to (3)$

$$\begin{bmatrix} -8 & -2 & -9 & 2 \\ 0 & \frac{5}{2} & \frac{5}{4} & \frac{5}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

System has a solution because there is no row of the type $[0 \dots 0 \; *]$, $* \neq 0$

So $\vec{u}$ is in Col(A).

To check $\vec{u} \in$ Nul(A) calculate

$$A\vec{u} = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Yes it is.

Problem 4.2 # 32

general polynomial in P_2

$$p = a + bt + ct^2$$

if $T(p) = 0$ then $\begin{bmatrix} p(0) \\ p(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} a \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow a = 0$

so all polynomials in ker T are

$bt + ct^2$

so $\ker(T) = \text{span}\{t, t^2\}$

so you can choose $p_1(t) = t$

$p_2(t) = t^2$

The range of T is the set of all vectors in $\mathbb{R}^2$ of the form $\begin{bmatrix} a \\ a \end{bmatrix}$

i.e., Range of $T = \text{span}\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$. 114

Problem 4.2 # 34

$T(f)$ is given by $T(f) = \int_0^x f(s)\, ds$.

$$T(f+g) = \int_0^x (f+g)(s)\, ds =$$

$$= \int_0^x (f(s) + g(s))\, ds = \int_0^x f(s)\, ds + \int_0^x g(s)\, ds = T(f) + T(g)$$

$$T(\lambda f) = \int_0^x (\lambda f)(s)\, ds = \int_0^x \lambda f(s)\, ds =$$

$\lambda \int_0^x f(s)\, ds = \lambda T(f)$ so it is linear

Ker is the set of functions such that $T(f) = 0$ (zero function) i.e.

$$\int_0^x f(s)\,ds = 0 \quad \text{for every } x$$

$\Rightarrow f(x) \equiv 0$ so only the zero function is in the kernel.