Problem 12.8 #12

Check stationary points.

\[ p_x = 2x = 0 \]
\[ p_y = 2y = 0 \]
\[ \rho = (0, 0) \]
\[ \rho(0) = 2(0) = 0 \]

on 4
\[ y = -1 \quad -1 \leq x \leq 3 \]
\[ p = x^2 + 1 \]

End points: \[ \begin{align*}
  x &= -1 & y &= -1 & p &= 2 \\
  x &= 3 & y &= -1 & p &= 10
\end{align*} \]

Stationary point: \[ \begin{align*}
  x &= 0 & y &= 0 & p &= 1
\end{align*} \]

On \( C \): \[ \begin{align*}
  x &= 3 & -1 \leq y \leq 4
\end{align*} \]

\[ p = 3 + y^2 \]

End points: \[ \begin{align*}
  x &= 3 & y &= -4 & p &= 10 \\
  x &= 3 & y &= 4 & p &= 25
\end{align*} \]

\[ \begin{align*}
  p &= 2 & y &= 0 \\
  x &= 3 & y &= 0 & p &= 9
\end{align*} \]
\[ g = 4 \quad -1 \leq x \leq 3 \]

\[ \phi = x^2 + 16 \]

End points:
- \( x = -1 \) \( y = 4 \) \( \phi = 17 \)
- \( x = 3 \) \( y = 4 \) \( \phi = 25 \)

Stationary point:
- \( x = 0 \) \( y = 4 \) \( \phi = 16 \)

Out:
- \( x = -1 \) \(-1 \leq y \leq 4\)
- \( \phi = 1 + y^2 \)

End points already considered.

Stationary point:
- \( \phi' = 2y = 0 \quad x = -1 \) \( y = 0 \) \( \phi = 1 \)
Minimum is \( (0,0) \) \( f = 0 \)

Maximum is \( (3,4) \) \( f = 25 \)

Problem 12.8 # 18

\[ \text{max } x + y + z \text{ with } \begin{align*}
  x &> 0 \\
  y &> 0 \\
  z &> 0
\end{align*} \]

\[ \text{with } x, y, z = V_0 \]

\[ f = \frac{V_0}{xy} \]

So we need to minimize \( f = x + y + \frac{V_0}{xy} \)
If it exists, must be a stationary point.

\[ p_x = 1 - \frac{V_0}{y} \frac{1}{x^2} = 0 \]

\[ p_y = 1 - \frac{V_0}{x} \frac{1}{y^2} = 0 \]

\[ 1 = \frac{V_0}{y} \frac{1}{x^2} \Rightarrow y = \frac{V_0}{x^2} \]

\[ l = \frac{V_0}{x} \frac{1}{y^2} \Rightarrow x = \frac{V_0}{y^2} \]
To see this: \[ x^2y = y^2x \implies x = y \]

and from 0 \[ x^3 = 1^0 \]

\[ x = y = 1^0 \frac{1}{3} \implies x = y^\frac{1}{3} \]

Check it is a local min. Use second partial test.

\[ P_{xx} = \frac{2}{y} \frac{1}{x^3} \quad P_{xy} = \frac{1}{x^2} \frac{1}{y} = \frac{1}{xy} \]

\[ P_{yy} = \frac{2}{y} \frac{1}{x^3} \]
\[ D = \begin{vmatrix} \frac{2}{V_0^{\frac{1}{2}}} & \frac{1}{V_0^{\frac{1}{3}}} \\ \frac{1}{V_0^{\frac{1}{2}}} & \frac{2}{V_0^{\frac{1}{3}}} \end{vmatrix} = \] 

\[ \frac{4}{V_0^{\frac{2}{2}}} - \frac{1}{V_0^{\frac{2}{3}}} = \frac{3}{V_0^{\frac{2}{3}}} > 0 \]

and \[ f_{xx} = \frac{2}{V_0^{\frac{1}{3}}} > 0 \]

So it is a local minimum.

By graphing the function one can see that this local minimum is in fact a global minimum.
distance \quad d = (x - 1)^2 + (y - 2)^2 + z^2

where \quad z^2 = x^2 + y^2

d = d(xy) = (x - 1)^2 + (y - 2)^2 + x^2 + y^2 = x^2 - 2x + 1 + y^2 - 4y + 4 + x^2 + y^2 = 2x^2 + 2y^2 - 2x - 4y + 5

calculate stationary points.

\dot{P}_x = 4x - 2 = 0 \quad x = \frac{1}{2}

\dot{P}_y = 4y - 4 = 0 \quad y = 1
at the point $P = \left( \frac{1}{2}, 1 \right)$

$$f = \frac{2}{2} \left( \frac{1}{2} \right)^2 + 2 \cdot \frac{1}{2} - \frac{1}{2} - 1 + 5 = \frac{5}{2}$$

$$2 - x - y + 5 = \frac{5}{2}$$

Prove that this is a min

[Completion of squares]

$$d(x,y) = \frac{2}{2} (x^2 - x) + 2 (y^2 - 2y) + 5$$

$$= 2 \left( x^2 - x + \frac{1}{4} \right) - \frac{1}{2} + 2 \left( y^2 - 2y + 1 \right) - 2 + 5$$

$$= 2 \left( x - \frac{1}{2} \right)^2 + 2 \left( y - 1 \right)^2 + \frac{5}{2} \geq \frac{5}{2}$$