HWK 5 Solutions

15-4 #22

a) Parametric curve with projection parallel to x axis

$$\begin{cases} z = x^2 y^3 \\ y = 2 \end{cases} \Rightarrow \begin{cases} x = t \\ y = 2 \\ z = 8t^2 \end{cases}$$

tangent vector $\vec{r}\,'(t) = \vec{i} + 16t\vec{k}$

when $t = 3$ $\quad \vec{r}\,'(3) = \vec{i} + 48\vec{k}$

line parallel to $\vec{r}\,'(3)$ and through $(3, 2, 7$

$$\boxed{\begin{aligned} x &= 3 + t \\ y &= 2 \\ z &= 72 + 48t \end{aligned}}$$

b) Parametric curve with projection parallel to y-axis

$$\begin{cases} z = x^2 y^3 \\ x = 3 \end{cases} \Rightarrow \begin{aligned} x &= 3 \\ y &= t \\ z &= 9t^3 \end{aligned}$$

$$\vec{r}'(t) = \vec{j} + 27t^2 \vec{k} \quad @ \quad t = 2$$

$$\vec{r}'(2) = \vec{j} + 108\vec{k}$$

line

$$\begin{aligned} x &= 3 \\ y &= 2 + t \\ z &= 72 + 108t \end{aligned}$$

c) Parametric curve with projection parallel

to $x = -y$

$$\begin{cases} z = x^2 y^3 \\ (x-3) = -(y-2) \end{cases} \Longrightarrow \begin{cases} x = t \\ y = -t+5 \\ z = t^2(5-t)^3 \end{cases}$$

$$\vec{r}\,'(t) = \vec{i} - \vec{j} + \left[2t(5-t)^3 - t^2 3(5-t)^2\right]\vec{u}$$

@ $t = 3$

$$\vec{r}\,'(3) = \vec{i} - \vec{j} - 60\vec{u}$$

line

$$\begin{cases} x = 3 + t \\ y = 2 - t \\ z = 72 - 60t \end{cases}$$

15.5 # 10

$$\nabla f_r = \cos x\, e^{y}\,\vec{i} + e^{y}\sin x\,\vec{j}$$

@ $\left(5\pi/6, 0\right)$

$$\nabla f_r = \cos\left(\frac{5\pi}{6}\right)\vec{i} + \sin\left(\frac{5\pi}{6}\right)\vec{j}$$

already a unit vector.

rate of change $|\nabla f_r| = 1$

15.6 #12

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$y e^{xy+z}\, 1 + x e^{xy+z}(-t) + e^{xy+z}\, 2t$$

In terms of s and t

$$(s-t)\,e^{(s+t)(s-t)+t^2} - (s+t)\,e^{(s+t)(s-t)+t^2} + e^{(s+t)(s-t)+t^2}$$

$$= \boxed{0}$$