HW #5 Solutions

3.2 #10

+1

+1

-1

$$A = \begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{vmatrix} \sim$$

$$\begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & -2 & 0 & 8 & -1 \\ 0 & -4 & 8 & 2 & 13 \end{vmatrix} \sim \begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & -4 & 7 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & -4 & 7 & -7 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = \text{REF}$$

$\det(\text{REF}) = -24$

$\det(A) = -\det(\text{REF}) = 24$

Problem 3.2 # 16

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$$

$$\det \begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix} = 3 \det \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 21$$

Problem 3.3 # 6

$$2x_1 + x_2 + x_3 = 4$$

$$-x_1 \qquad + 2x_3 = 2$$

$$3x_1 + x_2 + 3x_3 = -2$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 0 & 2 \\ -2 & 1 & 3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & -2 & 3 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & -2 \end{bmatrix}$$

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$$\det(A) = 6 - 1 - 4 + 3 = 4$$

$$\det(A_1) = -4 + 2 - 8 - 6 = -16$$

$$\det(A_2) = 12 + 24 + 2 - 6 + 8 + 12 = 52$$

$$\det(A_3) = 6 - 4 - 4 - 2 = -4$$

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{-16}{4} = -4$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{52}{4} = 13$$

$$x_3 = \frac{\det(A_3)}{\det(A)} = \frac{-4}{4} = -1$$

EXTRA CREDIT PROBLEM

a) $\vec{v}_1$ not in the span of $\{\vec{v}_2, \vec{v}_3\}$

b) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly independent

b) $\Rightarrow$ a) because if $\vec{v}_1$ was in the span of $\{\vec{v}_2, \vec{v}_3\}$ then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ would be linearly dependent.

However

a) $\not\Rightarrow$ b) consider $\vec{v}_2 = \vec{v}_3 = \vec{0}$ and $\vec{v}_1 \neq \vec{0}$ then $\vec{v}_1 \notin \operatorname{span}\{\vec{v}_2, \vec{v}_3\}$ but $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly dependent.

Conclusion a), b) are not the same thing.