

MATH 317 (Section A) Homework No. 5

Reading

Section 3.3, up to ‘The adjoint matrix’ (excluded)

Section 3.4

Suggested Problems

Section 3.3: Exercise 2 a) and 2 c), 22 a), b).

Section 3.4: Exercise 1 a), c), 3 a), c), 4 g) 5 e).

Problems to be handed in by Monday October 3-rd (in class)

Problem 1

Consider an $n \times n$ matrix A . Prove that the following facts are equivalent

1. A has a row which is a linear combination of other rows.
2. A has a column which is a linear combination of other columns.
- 3.

$$\det(A) = 0$$

Problem 2

Along with elementary row operations one can define elementary *column* operations of type *I*, *II* and *III*. In particular

1. An operation of the type *I*, multiplies a given column of A by a scalar c , i.e.

$$c(j) \rightarrow (j)$$

where (j) represents the j -th column of A .

2. An operation of the type *II*, multiplies the i -th column by a scalar c , sum the result with the j -th column and puts the final result in the j -th column, i.e.

$$c(i) + (j) \rightarrow (j)$$

3. An operation of type *III* swaps two columns, namely

$$(i) \leftrightarrow (j)$$

Each operation corresponds to multiplying the matrix A on the right by an appropriate elementary matrix, which we denote by C_I , C_{II} and C_{III} respectively.

- Find general expressions for the elementary matrices C_I , C_{II} and C_{III} .
- Give one 2×2 example for each elementary row operation.
- Determine in general how the three elementary column operations modify the determinant.

Problem 3 Problem 16, of Section 3.4.