HW #4 SOLUTIONS

Section 2.2 #20

$$(A - AX)^{-1} = X^{-1}B \qquad (3)$$

and $A - AX$, A, X invertible

a) $B = X(A-AX)^{-1}$

$$B(A-AX)X^{-1} = X(A-AX)^{-1}(A-AX)X^{-1}$$

$= I$

so B has an inverse given by $(A-AX)X^{-1}$, B is invertible.

b) Multiply (3) on the left by X we get

$$X(A-AX)^{-1} = B$$

multiply on the right by $(A-AX)$; we get.

$$X = B(A-AX) = BA - BAX$$

$$(I+BA)X = BA$$

$$X = (I+BA)^{-1}BA$$

$I+BA$ is invertible because.

$(I+BA) = BAX^{-1}$ is invertible since

$$(BAX^{-1})(XA^{-1}B^{-1}) = I$$

Section 2.2 # 32

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 2 & -2 & 2 & 0 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 0 & \times & \times & \times \end{array}\right]$$

The inverse does not exist since there is a row of all zeros.

Section 2.3 #14

A lower triangular matrix is inverti-
ble if and only if all the elements
on the main diagonal are different
from zero.

Proof if $a_{11}, \dots a_{nn} \neq 0$ (all)

then we can solve.

$$\begin{bmatrix} a_{11} & & \\ & \ddots & \\ * & & a_{nn} \end{bmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \overbrace{\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}}^{\vec{b}}$$

independently of the value of $\vec{b}$

first set $x_1 = \dfrac{b_1}{a_{11}}$

replace into second eqn, it gives

$$a_{12} \frac{b_1}{a_{11}} + a_{22} x_2 = b_2$$

$$\Rightarrow x_2 = + \frac{b_2}{a_{22}} - \frac{a_{12}}{a_{22}} \frac{b_1}{a_{11}}$$

and so on. Therefore the solution exist (and is unique) independently of the value of $\vec{b}$.

Moreover if one of the a_{jj}'s is zero there are values of $\vec{b}$ for which the eqns has no solution.

Consider $\bar{x}_1, \dots \bar{x}_{j-1}$ from previous steps as above. At the j-th step we obtain:

$$a_{j1}\bar{x}_1 + a_{j2}\bar{x}_2 + \dots + a_{jj-1}\bar{x}_{j-1} + 0\,x_j = b$$

so iff $b_j \neq a_{j1}\bar{x}_1 + a_{j2}\bar{x}_2 + \dots + a_{jj-1}\bar{x}_{j-1}$

the equation has no solution.

Therefore the system $A\vec{x} = \vec{b}$ does not have a solution for every $\vec{b}$ and A is not invertible.

$\square$

Section 2.4 # 8

$$\begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} X & Y & Z \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\begin{cases} AX = I & \Rightarrow X = A^{-1} \\ AY = 0 & Y = 0 \\ AZ + B = 0 & Z = -A^{-1}B \end{cases}$$

Assumption A is invertible square.

Section 2.4 # 10

$$\begin{bmatrix} I & 0 & 0 \\ C & I & 0 \\ A & B & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ Z & I & 0 \\ X & Y & I \end{bmatrix} =$$

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

$C + Z = 0$ $\qquad Z = -C$

$A + BZ + X = 0 \Rightarrow A + B(-C) = -X$

$\Rightarrow X = -A + BC$

$B + Y = 0$ $\qquad Y = -B$

So the inverse is

$$\begin{bmatrix} I & 0 & 0 \\ -C & I & 0 \\ -A+BC & -B & I \end{bmatrix}$$