# MATH SOLUTIONS #1

1.2 # 12

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \quad *$$

$$\sim \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{bmatrix} \sim \begin{array}{c} \begin{array}{ccccc} p & np & p & np & \phantom{0} \end{array} \\ \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Solution exists

matrix already in RREF $\quad x_2 = \lambda \quad x_4 = \mu.$

$$x_1 - 7\lambda + 6\mu = 5$$
$$x_3 - 2\mu = -3$$

$$\begin{cases} x_1 = 5 + 7\lambda - 6\mu \\ x_3 = -3 + 2\mu \\ x_2 = \lambda \\ x_4 = \mu \end{cases}$$

1.2 # 24

No it does not have a solution. Consider its RREF. There is one row where the leading term is in the last column. This row corresponds to an equation of the form

$0 = 1$ which is never satisfied.

1.3 # 12

Look at system $\vec{a}_1 x_1 + \vec{a}_2 x_2 + \vec{a}_3 x_3 = \vec{b}$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$ not consistent

$\vec{b}$ is NOT a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}$

1.4 # 16

Transform A in RREF

$$\begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -4 \\ 0 & -7 & -6 \\ 0 & +14 & 12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & -4 \\ 0 & -7 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

if after transformation $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ x \end{bmatrix}$ $x \neq 0$

there is no solution

To have a solution $\vec{b}$ must be a lin combination of the columns of $A$

$$\vec{b} = \lambda \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix} + \nu \begin{bmatrix} -4 \\ 6 \\ -8 \end{bmatrix}$$

1.4 # 32

No because call these vectors $\vec{a}_1$ $\vec{a}_2$ $\vec{a}_3$ then there should be for every $\vec{b}$ coefficients $x_1, x_2, x_3$ such that

$$\vec{a}_1 x_1 + \vec{a}_2 x_2 + \vec{a}_3 x_3 = \vec{b}$$

So the system $3 \times 4$

$A\vec{x} = \vec{b}$ will have to have a solution for every

...

A is $4 \times 3$ its RREF must have at least a row of zeros. because # of pivots is at most 3 i.e.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{etc.}$$

Same argument extends to general $n < m$

# HWK # 2 SOLUTIONS.

## Exercise 1.5 #16

$$\begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 2 & -6 & 6 \end{bmatrix}$$

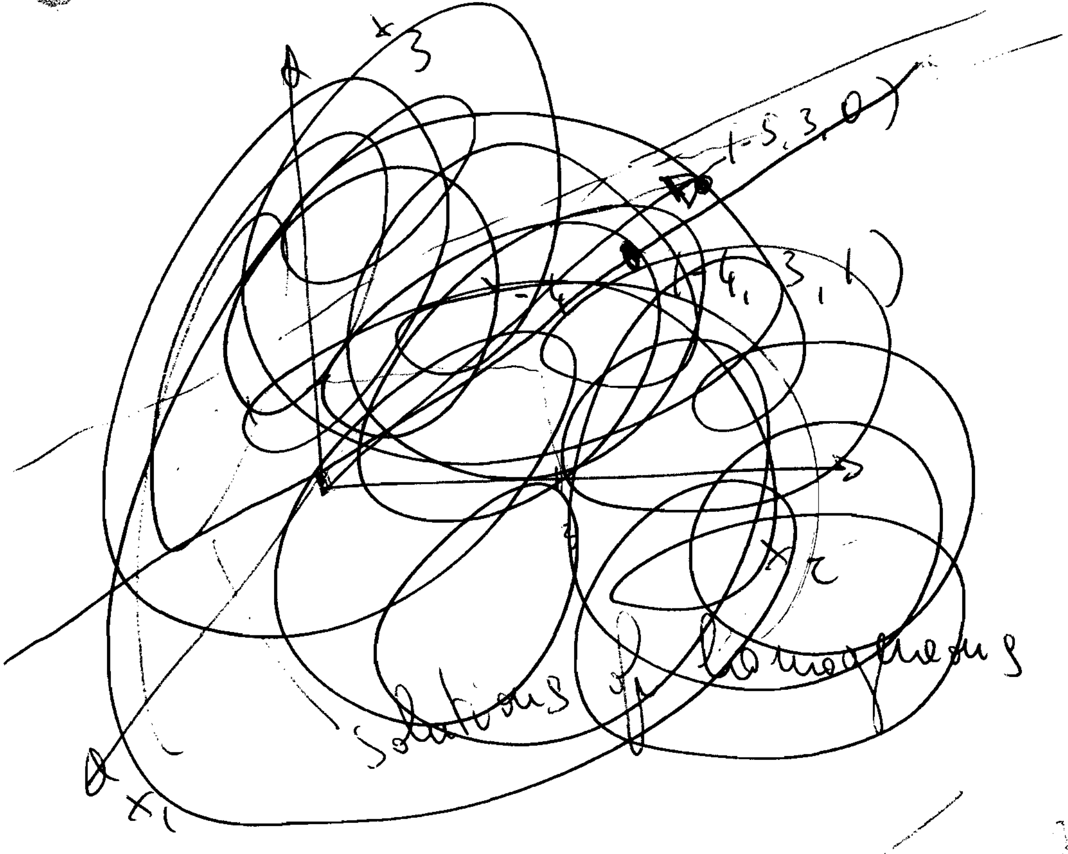
$$\sim \underbrace{\begin{bmatrix} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{REF}} \sim \underbrace{\begin{bmatrix} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{RREF}}$$

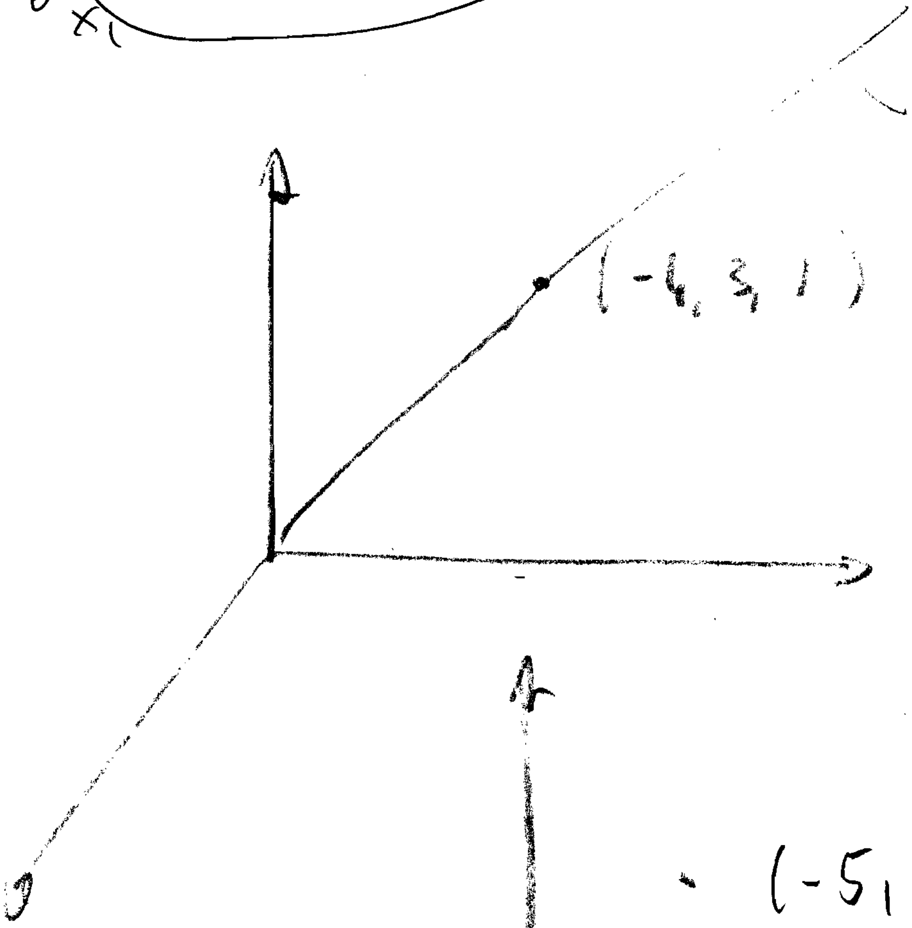
$x_3 = \lambda$

$$x_1 + 4\lambda = -5 \implies x_1 = -5 - 4\lambda$$

$$x_2 - 3\lambda = 3 \implies x_2 = 3 + 3\lambda.$$

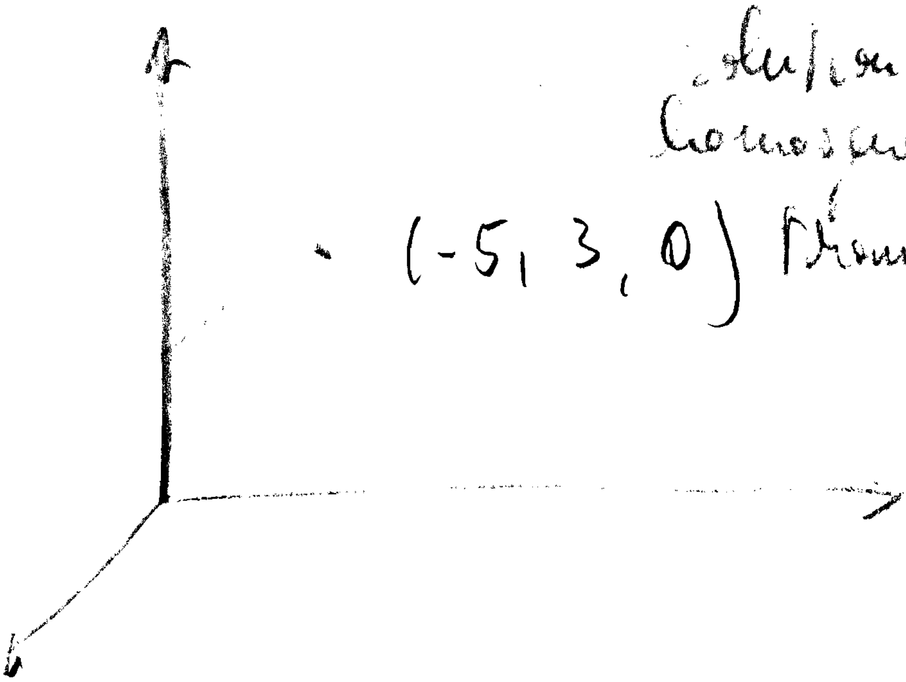
$$\vec{X}_0 = \underbrace{\begin{pmatrix} -5 \\ 3 \\ 0 \end{pmatrix}}_{\text{particular solution}} + \lambda \underbrace{\begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}}_{\text{general solution for homogeneous}}$$




solutions of homogeneous

solutions of inhomogeneous = solutions of homogeneous



(-5, 3, 0) translates

Exercise 1.7 # 6

$$A = \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

look at $A\vec{x} = 0$

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix} \sim \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 0 & -\frac{3}{4} & 3 \\ 0 & \frac{1}{4} & 6 \end{bmatrix} \sim$$

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 0 \\ 0 & \frac{1}{4} & 7 \end{bmatrix} \sim \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 0 & \frac{1}{4} & 7 \\ 0 & 0 & 0 \end{bmatrix}$$

~~[illegible]~~ exp.

Yes they are l. i.; every column is a pivot column.

EXERCISE 1.7 #14

$$\begin{bmatrix} 1 & -5 & 1 \\ -1 & 7 & 1 \\ -3 & 8 & h \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 1 \\ 0 & 2 & 2 \\ 0 & -7 & h+3 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & -5 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & h+3+7 \end{bmatrix}$$

l. i. iff $h+10 \neq 0$

l. d. iff $h=-10$

EXERCISE 1.7 #26

since $\{\vec{a}_1, \vec{a}_2\}$ is lin ind and $\vec{a}_3$ is not in span $\{\vec{a}_1, \vec{a}_2\}$ then $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ are lin. ind.

in fact if they were l. d. then 

$\lambda \vec{a}_3 + \mu \vec{a}_2 + \nu \vec{a}_1 = 0$ for at least one $\lambda, \mu, \nu \neq 0$

but if $\lambda \neq 0 \Rightarrow \vec{a}_3$ in span$\{\vec{a}_1, \vec{a}_2\}$
not true

so $\lambda = 0$ but $\{\vec{a}_1, \vec{a}_2\}$ are lin ind

so $\mu = 0$, $\nu = 0$ $\boxed{}$

$$[\vec{a}_1\ \vec{a}_2\ \vec{a}_3] \sim \begin{bmatrix} \boxed{} & * & * \\ 0 & \boxed{} & * \\ 0 & 0 & \boxed{} \\ 0 & 0 & 0 \end{bmatrix}$$ REF

in RREF we have $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

HWK # 3 Solutions

Section 1.9



# 18

$T(x_1, x_2) = (2x_2 - 3x_1,\ x_1 - 4x_2,\ 0,\ x_2)$

$$A = \begin{bmatrix} -3 & 2 \\ 1 & -4 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

# 26 The matrix associated to $T: \mathbb{R}^3 \to \mathbb{R}^2$ is $A = \begin{bmatrix} 1 & 4 & -5 \\ 3 & -7 & 4 \end{bmatrix}$

Not one to one because at least one column of A must be a non pivot column so $A\vec{x} = \vec{0}$ has non trivial solution.

Calculate REF of A

$\sim \begin{bmatrix} 1 & 4 & -5 \\ 0 & -13 & -9 \end{bmatrix}$

every row has a pivot so every ~~sola~~ system $A\vec{x} = \vec{b}$ has a solution.

T is onto

#.34 Assume $T: \mathbb{R}^n \to \mathbb{R}^m$

Because the definition of onto says that T is onto if and only if for every $\vec{b}$ in $\mathbb{R}^m$ there exists $\vec{x}$ in $\mathbb{R}^n$ such that $T(\vec{x}) = \vec{b}$

Section 2.1

#8 3 rows matrix:

# of rows of BC = # of rows of B

# of columns of BC = # of columns of C

#28

$u^T v = v^T u$

$u^T v$ is a number. If I transpose it I obtain itself but

$u^T v = (u^T v)^T = v^T (u^T)^T = v^T u$

$u v^T = (v u^T)^T$ (1)

$uv^T$ is $n \times n$

we have

$$(vu^T)^T = (u^T)^T v^T = uv^T$$