HWK # 11 Solutions.

Pr 6.3 #4

$\vec{u}_1 \cdot \vec{u}_2 = 3 \cdot (-4) + 4 \cdot 3 = 0$ ✓

$$\operatorname{proj} y = \frac{\vec{y} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} \vec{u}_1 + \frac{\vec{y} \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} \vec{u}_2 =$$

$$\frac{\begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}}{\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}} \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + \frac{\begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix}}{\begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix}} \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} =$$

$$\frac{30}{25} \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + \frac{-15}{25} \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{18}{5} \\ \frac{24}{5} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{12}{5} \\ -\frac{9}{5} \\ 0 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix}}$$

Pr 6.3 # 16

if $\hat{y}$ is the projection onto the subspace spanned by $\vec{v}_1$ and $\vec{v}_2$ then.

$\|y - \hat{y}\|$ is the distance.

Find projection $\hat{y}$

$$\hat{y} = \frac{\vec{y} \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1 + \frac{\vec{y} \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2} \vec{v}_2 =$$

$$\frac{\begin{pmatrix} 3 \\ -1 \\ 1 \\ 13 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix}} \begin{pmatrix} 1 \\ -2 \\ -1 \\ 2 \end{pmatrix} + \frac{\begin{pmatrix} 3 \\ -1 \\ 1 \\ 13 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix}}{\begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix}} \begin{pmatrix} -4 \\ 1 \\ 0 \\ 3 \end{pmatrix}$$

$$= \frac{30}{10}\begin{pmatrix}1\\-2\\-1\\2\end{pmatrix} + \frac{26}{26}\begin{pmatrix}-4\\1\\0\\3\end{pmatrix} = \begin{pmatrix}3\\-6\\-3\\6\end{pmatrix} + \begin{pmatrix}-4\\1\\0\\3\end{pmatrix} =$$

$$\begin{pmatrix}-1\\-5\\-3\\8\end{pmatrix} = \hat{y}$$

$$(y-\hat{y}) = \begin{pmatrix}3\\-1\\1\\13\end{pmatrix} - \begin{pmatrix}-1\\-5\\-3\\8\end{pmatrix} = \begin{pmatrix}4\\4\\4\\22\end{pmatrix}$$

$$\| y - \hat{y} \| = \sqrt{4^2 + 4^2 + 4^2 + 22^2} = \sqrt{582} =$$

Pr 6.4 #10

Apply Gram-Schmidt.

$$\vec{v}_1 = \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix} 6 \\ -8 \\ -2 \\ -4 \end{pmatrix} - \frac{\begin{pmatrix} 6 \\ -8 \\ -2 \\ -4 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix}} \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} 6 \\ -8 \\ -2 \\ -4 \end{pmatrix} - \frac{(-36)}{12} \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ -2 \\ -4 \end{pmatrix} + \begin{pmatrix} -3 \\ 9 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\vec{v}_3 = \begin{pmatrix} 6 \\ 3 \\ 6 \\ -3 \end{pmatrix} - \frac{\begin{pmatrix} 6 \\ 3 \\ 6 \\ -3 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix}} \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 6 \\ 3 \\ 6 \\ -3 \end{pmatrix} \circ \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}}{\begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix} \circ \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}} \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 6 \\ 3 \\ 6 \\ -3 \end{pmatrix} - \frac{6}{12}\begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix} - \frac{30}{12}\begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix} =$$

$$\begin{pmatrix} 6 + \frac{1}{2} - \frac{15}{2} \\ 3 - \frac{3}{2} - \frac{5}{2} \\ 6 - \frac{1}{2} - \frac{5}{2} \\ -3 - \frac{1}{2} + \frac{5}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 3 \\ -1 \end{pmatrix}$$

$$\vec{v}_1 = \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix} \quad \vec{v}_3 = \begin{pmatrix} -1 \\ -1 \\ 3 \\ -1 \end{pmatrix}$$

Pr 6.4 #16

Do Gram-Schmidt.

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{pmatrix} - \frac{\begin{pmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{pmatrix} - \frac{16}{4} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{v}_3 = \begin{pmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{pmatrix} - \frac{\begin{pmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}}{\begin{pmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}} \begin{pmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix}5\\1\\3\\2\\8\end{pmatrix} - \frac{14}{4}\begin{pmatrix}1\\-1\\0\\1\\1\end{pmatrix} - \frac{12}{8}\begin{pmatrix}-1\\1\\2\\1\\1\end{pmatrix} =$$

$$\begin{pmatrix}5 - \frac{7}{2} + \frac{3}{2}\\ 1 + \frac{7}{2} - \frac{3}{2}\\ 3 - 3\\ 2 - \frac{7}{2} - \frac{3}{2}\\ 8 - \frac{7}{2} - \frac{3}{2}\end{pmatrix} = \begin{pmatrix}3\\3\\0\\-3\\3\end{pmatrix}$$

$$A = \begin{bmatrix}1 & -1 & 3\\ -1 & 1 & 3\\ 0 & 2 & 0\\ 1 & 1 & -3\\ 1 & 1 & 3\end{bmatrix}\begin{bmatrix}1 & 4 & \frac{7}{2}\\ 0 & 1 & \frac{3}{2}\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 3 & 5\\ -1 & -3 & 1\\ 0 & 2 & 3\\ 1 & 3 & 2\\ 1 & 5 & 8\end{bmatrix}$$

Normalization

$$\|\vec{v}_1\| = 2$$

$$\|\vec{v}_2\| = \sqrt{1+1+2^2+1+1} = \sqrt{8}$$

$$\|\vec{v}_3\| = \sqrt{9+9+9+9} = \sqrt{36} = 6$$

$$Q_2 = \begin{bmatrix} \frac{1}{2} & -\frac{1}{\sqrt{8}} & \frac{1}{2} \\ -\frac{1}{2} & 1/\sqrt{8} & \frac{1}{2} \\ 0 & 2/\sqrt{8} & 0 \\ \frac{1}{2} & 1/\sqrt{8} & -\frac{1}{2} \\ \frac{1}{2} & 1/\sqrt{8} & \frac{1}{2} \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 2 & 8 & 7 \\ 0 & \sqrt{8} & \frac{3}{2}\sqrt{8} \\ 0 & 0 & 6 \end{bmatrix}$$

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Pr 6.5 # 10.

Columns for A are already orthogonal

$$\operatorname{proj}\hat{b} = \frac{\begin{pmatrix}3\\-1\\5\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\1\end{pmatrix}}{\begin{pmatrix}1\\-1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\1\end{pmatrix}}\begin{pmatrix}1\\-1\\1\end{pmatrix} + \frac{\begin{pmatrix}3\\-1\\5\end{pmatrix}\cdot\begin{pmatrix}2\\4\\2\end{pmatrix}}{\begin{pmatrix}2\\4\\2\end{pmatrix}\cdot\begin{pmatrix}2\\4\\2\end{pmatrix}}\begin{pmatrix}2\\4\\2\end{pmatrix}$$

$$\frac{9}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix} + \frac{12}{24}\begin{pmatrix}2\\4\\2\end{pmatrix} = \begin{pmatrix}3+1\\-3+2\\3+1\end{pmatrix} = \begin{pmatrix}4\\-1\\4\end{pmatrix}$$

Solve

$$A\vec{x} = \hat{b} \qquad \begin{bmatrix}1 & 2\\-1 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}4\\-1\\4\end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ -1 & 4 & -1 \\ 1 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$6x_2 = 3$ $\qquad$ $x_2 = \frac{1}{2}$

$x_1 + 1 = 4 \Rightarrow x_1 = 3$

least square solution

$$\begin{pmatrix} 3 \\ \frac{1}{2} \end{pmatrix}$$

Alternatively.

$$A^T A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 24 \end{pmatrix}$$

11b

$$A^T \vec{b} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

$$3x_1 = 9 \quad \Rightarrow \quad x_1 = 3$$

$$24\, x_2 = 12 \quad \Rightarrow \quad x_2 = \frac{1}{2}$$