

## MATH 267 Section E1 Final No. 2 Practice

**Problem 1**(7 points)

a) Given a function  $f(t)$  give the definition of the Laplace transform of  $f(t)$ ,  $F(s)$ .

b)  $F(s)$  is defined only for certain values of  $s$ . For example, if  $f(t) = e^{2t}$ , then  $F(s) = \frac{1}{s-2}$ , is defined only for  $s > 2$ . Why?

**Problem 2**(7 points) Consider the discontinuous function

$$f(t) = t, \quad 0 \leq t < 1,$$

$$f(t) = e^t, \quad 1 \leq t.$$

Describe this function using the step function and calculate its Laplace transform.

**Problem 3** (7 points) Calculate the inverse Laplace transform of the function

$$\frac{e^{-2s}}{s^2 - 1}.$$

**Problem 4**(7 points) Solve the boundary value problem

$$y' - 4y = \delta(t - 2), \quad y(0) = 1.$$

**Problem 5**(7 points) Calculate the inverse Laplace transform of the function  $\frac{1}{s^3}$ , knowing the following three things ONLY

- 1) The inverse transform of  $\frac{1}{s}$  is 1.
- 2) The inverse transform of  $\frac{1}{s^2}$  is  $t$ .
- 3) The convolution theorem.

**Problem 6** (7 points)

1) Illustrate two properties of the exponential of a matrix  $e^{At}$  which are extensions of analogous properties of the scalar exponential function.

2) Explain the difference between a fundamental matrix  $\Psi(t)$  and the exponential matrix  $e^{At}$ .

**Problem 7** (7 points) Consider the time varying system

$$\dot{x}_1 = tx_2,$$

$$\dot{x}_2 = tx_2.$$

Verify that the two functions

$$\vec{x} = \begin{pmatrix} e^{\frac{1}{2}t^2} \\ e^{\frac{1}{2}t^2} \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

are a fundamental set of solutions for this system.