Problem 1 (7 points)
   a) Given a function $f(t)$ give the definition of the Laplace transform of $f(t)$, $F(s)$.
   b) $F(s)$ is defined only for certain values of $s$. For example, if $f(t) = e^{2t}$, then $F(s) = \frac{1}{s-2}$, is defined only for $s > 2$. Why?

Problem 2 (7 points) Consider the discontinuous function

\[ f(t) = t, \quad 0 \leq t < 1, \]
\[ f(t) = e^t, \quad 1 \leq t. \]

Describe this function using the step function and calculate its Laplace transform.

Problem 3 (7 points) Calculate the inverse Laplace transform of the function

\[ \frac{e^{-2s}}{s^2 - 1}. \]

Problem 4 (7 points) Solve the boundary value problem

\[ y' - 4y = \delta(t - 2), \quad y(0) = 1. \]

Problem 5 (7 points) Calculate the inverse Laplace transform of the function $\frac{1}{s^3}$, knowing the following three things ONLY

1) The inverse transform of $\frac{1}{s}$ is 1.
2) The inverse transform of $\frac{1}{s^2}$ is $t$.
3) The convolution theorem.

Problem 6 (7 points)

1) Illustrate two properties of the exponential of a matrix $e^{At}$ which are extensions of analogous properties of the scalar exponential function.
2) Explain the difference between a fundamental matrix $\Psi(t)$ and the exponential matrix $e^{At}$.

Problem 7 (7 points) Consider the time varying system

\[ \dot{x}_1 = tx_2, \]
\[ \dot{x}_2 = tx_2. \]

Verify that the two functions

\[ \vec{x} = \begin{pmatrix} e^{\frac{1}{2}t^2} \\ e^{\frac{1}{2}t^2} \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \]

are a fundamental set of solutions for this system.