Determine the phase velocity and the wavelength, and then sketch \( y(x,t) \) at \( t = 2s \) over the range from \( x = 0 \) to \( x = 2\lambda \).

1.4 Two waves on a string are given by the following functions:

\[
y_1(x,t) = 3 \cos(20t - 30x) \quad \text{cm},
\]

\[
y_2(x,t) = -3 \cos(20t + 30x) \quad \text{cm},
\]

where \( x \) is in centimeters. The waves are said to interfere constructively when their superposition \( y_s = y_1 + y_2 \) is a maximum and they interfere destructively when \( y_s \) is a minimum.

(a) What are the directions of propagation of waves \( y_1(x,t) \) and \( y_2(x,t) \)?

(b) At \( t = (\pi/50) \) s, at what location \( x \) do the two waves interfere constructively, and what is the corresponding value of \( y_s \)?

(c) At \( t = (\pi/50) \) s, at what location \( x \) do the two waves interfere destructively, and what is the corresponding value of \( y_s \)?

1.5* An oscillator that generates a sinusoidal wave on a string completes 20 vibrations in 30 s. The wave peak is observed to travel a distance of 2.8 m along the string in 5 s. What is the wavelength?

1.6 Give expressions for \( y(x,t) \) for a sinusoidal wave traveling along a string in the negative \( x \)-direction, given that \( y_{\text{max}} = 20 \text{ cm}, \lambda = 30 \text{ cm}, f = 5 \text{ Hz}, \) and

(a) \( y(x,0) = 0 \) at \( x = 0 \),

(b) \( y(x,0) = 0 \) at \( x = 7.5 \text{ cm} \).

1.7* Given two waves characterized by

\[
y_1(t) = 6 \cos \omega t,
\]

\[
y_2(t) = 6 \sin(\omega t + 30^\circ),
\]

does \( y_2(t) \) lead or lag \( y_1(t) \) and by what phase angle?

1.8 The voltage of an electromagnetic wave traveling on a transmission line is given by

\[
v(z,t) = 3e^{-\alpha z} \sin(2\pi \times 10^6 t - 10\pi z) \quad \text{V},
\]

where \( z \) is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.

(b) At \( z = 2 \text{ m} \), the amplitude of the wave was measured to be 1 V. Find \( \alpha \).

Section 1.5: Complex Numbers

1.9* Complex numbers \( z_1 \) and \( z_2 \) are given by

\[
z_1 = 3 - j2,
\]

\[
z_2 = -4 + j2.
\]

(a) Express \( z_1 \) and \( z_2 \) in polar form.

(b) Find \( |z_1| \) by applying Eq. (1.41) and again by applying Eq. (1.43).

(c) Determine the product \( z_1z_2 \) in polar form.

(d) Determine the ratio \( z_1/z_2 \) in polar form.

(e) Determine \( z_1^2 \) in polar form.

1.10 Complex numbers \( z_1 \) and \( z_2 \) are given by

\[
z_1 = 5 \angle -60^\circ,
\]

\[
z_2 = 2 \angle 45^\circ.
\]

(a) Determine the product \( z_1z_2 \) in polar form.

(b) Determine the product \( z_1^2 \) in polar form.

(c) Determine the ratio \( z_1/z_2 \) in polar form.

(d) Determine the ratio \( z_1^2/z_2^2 \) in polar form.

(e) Determine \( \sqrt{z_1} \) in polar form.

1.11* If \( z = 3 - j4 \), find the value of \( \ln(z) \).

1.12 If \( z = 3 - j4 \), find the value of \( e^{zt} \).

Section 1.6: Phasors

1.13* A voltage source given by

\[
v_s(t) = 10\cos(2\pi \times 10^3 t - 30^\circ) \quad \text{V}
\]

is connected to a series RC load as shown in Fig. 1-19. If \( R = 1 \text{ M\Omega} \) and \( C = 100 \text{ pF} \), obtain an expression for \( v_c(t) \), the voltage across the capacitor.

1.14 Find the phasors of the following time functions:
The Laplacian of a scalar function is defined as the divergence of the gradient of that function.

GLOSSARY OF IMPORTANT TERMS

Provide definitions or explain the meaning of the following terms:

- scalar quantity
- vector quantity
- magnitude
- unit vector
- base vectors
- position vector
- distance vector
- simple product
- scalar (or dot) product
- vector (or cross) product
- orthogonal coordinate system
- Cartesian coordinate system
- cylindrical coordinate system
- spherical coordinate system
- radial distance \( r \)
- azimuth angle \( \phi \)
- zenith angle \( \theta \)
- range \( R \)
- gradient operator
- directional derivative
- flux lines
- flux density
- divergence operator
- solenoidal field
- divergence theorem
- circulation of a vector
- curl operator
- conservative field
- Stokes’s theorem
- Laplacian operator

PROBLEMS

Section 3-1: Vector Algebra

3.1* In Cartesian coordinates, the three corners of a triangle are \( P_1(0,2,2) \), \( P_2(2,-2,2) \), and \( P_3(1,1,-2) \). Find the area of the triangle.

3.2 Given \( \mathbf{A} = \hat{x}2 - \hat{y}3 + \hat{z}1 \) and \( \mathbf{B} = \hat{x}B_x + \hat{y}2 + \hat{z}B_z \):
   (a) find \( B_x \) and \( B_z \) if \( \mathbf{A} \) is parallel to \( \mathbf{B} \);
   (b) find a relation between \( B_x \) and \( B_z \) if \( \mathbf{A} \) is perpendicular to \( \mathbf{B} \).

3.3* Given vectors \( \mathbf{A} = \hat{x} + \hat{y}2 - \hat{z}3 \), \( \mathbf{B} = \hat{x}3 - \hat{y}4 \), and \( \mathbf{C} = \hat{y}3 - \hat{z}4 \), find
   (a) \( \mathbf{A} \) and \( \hat{a} \),
   (b) the component of \( \mathbf{B} \) along \( \mathbf{C} \),
   (c) \( \theta_{AC} \),
   (d) \( \mathbf{A} \times \mathbf{C} \),
   (e) \( \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) \),
   (f) \( \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \),
   (g) \( \hat{x} \times \mathbf{B} \), and
   (h) \( (\mathbf{A} \times \hat{y}) \cdot \hat{z} \).

3.4 Given vectors \( \mathbf{A} = \hat{x}2 - \hat{y}3 + \hat{z}1 \) and \( \mathbf{B} = \hat{x}3 - \hat{z}2 \), find a vector \( \mathbf{C} \) whose magnitude is 6 and whose direction is perpendicular to both \( \mathbf{A} \) and \( \mathbf{B} \).

3.5* Given \( \mathbf{A} = \hat{x}(2x + 3y) - \hat{y}(2y + 3x) + \hat{z}(3x - y) \), determine a unit vector parallel to \( \mathbf{A} \) at point \( P(1, -1, 2) \).

3.6 By expansion in Cartesian coordinates, prove
   (a) the relation for the scalar triple product given by Eq. (3.29), and
   (b) the relation for the vector triple product given by Eq. (3.33).

*Answer(s) available in Appendix D.
Sections 3-2 and 3-3: Coordinate Systems

3.7* Convert the coordinates of the following points from Cartesian to cylindrical and spherical coordinates:
(a) \( P_1(1,2,0) \),
(b) \( P_2(0,0,3) \),
(c) \( P_3(1,1,2) \), and
(d) \( P_4(-3,3,-3) \).

3.8 Use the appropriate expression for the differential surface area \( ds \) to determine the area of each of the following surfaces:
(a) \( r = 3; \ 0 \leq \phi \leq \pi/3; \ -2 \leq z \leq 2 \),
(b) \( 2 \leq r \leq 5; \ \pi/2 \leq \phi \leq \pi; \ z = 0 \),
(c) \( 2 \leq r \leq 5; \ \phi = \pi/4; \ -2 \leq z \leq 2 \),
(d) \( R = 2; \ 0 \leq \theta \leq \pi/3; \ 0 \leq \phi \leq \pi, \) and
(e) \( 0 \leq R \leq 5; \ \theta = \pi/3; \ 0 \leq \phi \leq 2\pi \).
Also sketch the outline of each surface.

3.9* Find the volumes described by
(a) \( 2 \leq r \leq 5; \ \pi/2 \leq \phi \leq \pi; \ 0 \leq z \leq 2 \), and
(b) \( 0 \leq R \leq 5; \ 0 \leq \theta \leq \pi/3; \ 0 \leq \phi \leq 2\pi \).
Also sketch the outline of each volume.

3.10 Given vectors
\[
A = \hat{r}(\cos\phi + 3z) - \hat{\theta}(2r + 4\sin\phi) + \hat{z}(r - 2z),
\]
\[
B = -\hat{r}\sin\phi + \hat{\phi}\cos\phi,
\]
find
(a) \( \theta_{AB} \) at \( (2,\pi/2,0) \), and
(b) a unit vector perpendicular to both \( A \) and \( B \) at \( (2,\pi/3,1) \).

3.11* Determine the distance between the following pairs of points:
(a) \( P_1(1,1,2) \) and \( P_2(0,2,2) \),
(b) \( P_3(2,\pi/3,1) \) and \( P_4(4,\pi/2,0) \), and
(c) \( P_5(3,\pi,\pi/2) \) and \( P_6(4,\pi/2,\pi) \).

3.12 Transform the following vectors into cylindrical coordinates and then evaluate them at the indicated points:
(a) \( A = \hat{r}(x+y) \) at \( P_1(1,2,3) \),
(b) \( B = \hat{\theta}(y-x) + \hat{\phi}(x-y) \) at \( P_2(1,0,2) \),
(c) \( C = \hat{r}y^2/(x^2 + y^2) - \hat{\theta}x^2/(x^2 + y^2) + \hat{z}4 \) at \( P_3(1,-1,2) \), and
(d) \( D = \hat{R}\sin\theta + \hat{\theta}\cos\theta + \hat{\phi}\cos^2\phi \) at \( P_4(2,\pi/2,\pi/4) \).

3.13* Transform the following vectors into spherical coordinates and then evaluate them at the indicated points:
(a) \( A = \hat{r}y^2 + \hat{\theta}xz + \hat{z}4 \) at \( P_1(1,-1,2) \),
(b) \( B = \hat{\phi}(x^2 + y^2 + z^2) - \hat{\theta}(x^2 + y^2) \) at \( P_2(-1,0,2) \), and
(c) \( C = \hat{r}\cos\phi - \hat{\phi}\sin\phi + \hat{\phi}\cos^2\phi\sin\phi \) at \( P_3(2,\pi/4,2) \).

Sections 3-4 to 3-7: Gradient, Divergence, and Curl Operators

3.14 Find the gradient of the following scalar functions:
(a) \( T = 2/(x^2 + z^2) \),
(b) \( V = xy^2z^3 \),
(c) \( U = z\cos\phi/(1 + r^2) \), and
(d) \( W = e^{-R}\sin\theta \).

3.15 Follow a procedure similar to that leading to Eq. (3.82) to derive the expression given by Eq. (3.83) for \( V \) in spherical coordinates.

3.16 For the scalar function \( V = xy - z^2 \), determine its directional derivative along the direction of vector \( A = (\hat{x} - \hat{y}z) \) and then evaluate it at \( P(1,-1,2) \).

3.17* For the vector field \( E = \hat{r}xz - \hat{\theta}yz^2 - \hat{\phi}xy \), verify the divergence theorem by computing
(a) the total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each and parallel to the Cartesian axes, and
(b) the integral of $\nabla \cdot E$ over the cube's volume.

3.18 For the vector field $E = \hat{r}10e^{-r} - z\hat{z}$, verify the divergence theorem for the cylindrical region enclosed by $r = 2$, $z = 0$, and $z = 4$.

3.19* A vector field $D = \hat{r}r^3$ exists in the region between two concentric cylindrical surfaces defined by $r = 1$ and $r = 2$, with both cylinders extending between $z = 0$ and $z = 5$. Verify the divergence theorem by evaluating
(a) $\int_S D \cdot ds$, and
(b) $\int_V \nabla \cdot D \, dv$.

3.20 For the vector field $D = \hat{R}3R^2$, evaluate both sides of the divergence theorem for the region enclosed between the spherical shells defined by $R = 1$ and $R = 2$.

3.21* For the vector field $E = \hat{x}xy - \hat{y}(x^2 + 2y^2)$, calculate
(a) $\int_C E \cdot dl$ around the triangular contour shown in Fig. 3-25(a), and
(b) $\int_S (\nabla \times E) \cdot ds$ over the area of the triangle.

3.22 Repeat Problem 3.21 for the contour shown in Fig. 3-25(b).

3.23* Verify Stokes's theorem for the vector field
$$B = (\hat{r}r\cos\phi + \hat{\phi}\sin\phi)$$
by evaluating

(a) $\oint_C B \cdot dl$ over the semicircular contour shown in Fig. 3-26(a), and
(b) $\int_S (\nabla \times B) \cdot ds$ over the surface of the semicircle.

3.24 Repeat Problem 3.23 for the contour shown in Fig. 3-26(b).

3.25* Determine if each of the following vector fields is solenoidal, conservative, or both:
(a) $A = \hat{x}2xy - \hat{y}y^2$,
(b) $B = \hat{x}x^2 - \hat{y}y^2 + \hat{z}2z$,
(c) $C = \hat{r}(\sin\phi)/r^2 + \hat{\phi}(\cos\phi)/r^2$,
(d) $D = \hat{R}/R$. 
3.26 Find the Laplacian of the following scalar functions:

(a) \( V = x^2yz^3 \),
(b) \( V = xy + yz + zx \),
(c) \( V = 1/(x^2 + y^2) \),
(d) \( V = 5e^{-r}\cos\phi \),
(e) \( V = 10e^{-R}\sin\theta \).