Determine the phase velocity and the wavelength, and then sketch $y(x,t)$ at $t = 2s$ over the range from $x = 0$ to $x = 2\lambda$.

1.4 Two waves on a string are given by the following functions:

$$y_1(x,t) = 3\cos(20t - 30x) \qquad \text{(cm)},$$
$$y_2(x,t) = -3\cos(20t + 30x) \qquad \text{(cm)},$$

where x is in centimeters. The waves are said to interfere constructively when their superposition $y_s = y_1 + y_2$ is a maximum and they interfere destructively when y_s is a minimum.

(a) What are the directions of propagation of waves $y_1(x,t)$ and $y_2(x,t)$?
(b) At $t = (\pi/50)$ s, at what location x do the two waves interfere constructively, and what is the corresponding value of y_s?
(c) At $t = (\pi/50)$ s, at what location x do the two waves interfere destructively, and what is the corresponding value of y_s?

1.5* An oscillator that generates a sinusoidal wave on a string completes 20 vibrations in 30 s. The wave peak is observed to travel a distance of 2.8 m along the string in 5 s. What is the wavelength?

1.6 Give expressions for $y(x,t)$ for a sinusoidal wave traveling along a string in the negative x-direction, given that $y_{max} = 20$ cm, $\lambda = 30$ cm, $f = 5$ Hz, and
(a) $y(x,0) = 0$ at $x = 0$,
(b) $y(x,0) = 0$ at $x = 7.5$ cm.

1.7* Given two waves characterized by

$$y_1(t) = 6\cos\omega t,$$
$$y_2(t) = 6\sin(\omega t + 30°),$$

does $y_2(t)$ lead or lag $y_1(t)$ and by what phase angle?

1.8 The voltage of an electromagnetic wave traveling on a transmission line is given by

$$v(z,t) = 3e^{-\alpha z}\sin(2\pi \times 10^9 t - 10\pi z) \qquad \text{(V)},$$

where z is the distance in meters from the generator.
(a) Find the frequency, wavelength, and phase velocity of the wave.
(b) At $z = 2$ m, the amplitude of the wave was measured to be 1 V. Find α.

Section 1-5: Complex Numbers

1.9* Complex numbers z_1 and z_2 are given by

$$z_1 = 3 - j2,$$
$$z_2 = -4 + j2.$$

(a) Express z_1 and z_2 in polar form.
(b) Find $|z_1|$ by applying Eq. (1.41) and again by applying Eq. (1.43).
(c) Determine the product $z_1 z_2$ in polar form.
(d) Determine the ratio z_1/z_2 in polar form.
(e) Determine z_1^3 in polar form.

1.10 Complex numbers z_1 and z_2 are given by

$$z_1 = 5\angle{-60°},$$
$$z_2 = 2\angle{45°}.$$

(a) Determine the product $z_1 z_2$ in polar form.
(b) Determine the product $z_1 z_2^*$ in polar form.
(c) Determine the ratio z_1/z_2 in polar form.
(d) Determine the ratio z_1^*/z_2^* in polar form.
(e) Determine $\sqrt{z_1}$ in polar form.

1.11* If $z = 3 - j4$, find the value of $\ln(z)$.

1.12 If $z = 3 - j4$, find the value of e^z.

Section 1-6: Phasors

1.13* A voltage source given by

$$v_s(t) = 10\cos(2\pi \times 10^3 t - 30°) \qquad \text{(V)}$$

is connected to a series RC load as shown in Fig. 1-19. If $R = 1$ MΩ and $C = 100$ pF, obtain an expression for $v_c(t)$, the voltage across the capacitor.

1.14 Find the phasors of the following time functions:

- The Laplacian of a scalar function is defined as the divergence of the gradient of that function.

GLOSSARY OF IMPORTANT TERMS

Provide definitions or explain the meaning of the following terms:

scalar quantity
vector quantity
magnitude
unit vector
base vectors
position vector
distance vector
simple product
scalar (or dot) product
vector (or cross) product
orthogonal coordinate system
Cartesian coordinate system
cylindrical coordinate system
spherical coordinate system
radial distance r
azimuth angle ϕ
zenith angle θ
range R
gradient operator
directional derivative
flux lines
flux density
divergence operator
solenoidal field
divergence theorem
circulation of a vector
curl operator
conservative field
Stokes's theorem
Laplacian operator

PROBLEMS

Section 3-1: Vector Algebra

3.1* In Cartesian coordinates, the three corners of a triangle are $P_1(0,2,2)$, $P_2(2,-2,2)$, and $P_3(1,1,-2)$. Find the area of the triangle.

3.2 Given $\mathbf{A} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}3 + \hat{\mathbf{z}}1$ and $\mathbf{B} = \hat{\mathbf{x}}B_x + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}B_z$:

(a) find B_x and B_z if $\mathbf{A}$ is parallel to $\mathbf{B}$;

(b) find a relation between B_x and B_z if $\mathbf{A}$ is perpendicular to $\mathbf{B}$.

3.3* Given vectors $\mathbf{A} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3$, $\mathbf{B} = \hat{\mathbf{x}}3 - \hat{\mathbf{y}}4$, and $\mathbf{C} = \hat{\mathbf{y}}3 - \hat{\mathbf{z}}4$, find

(a) A and $\hat{\mathbf{a}}$,

(b) the component of $\mathbf{B}$ along $\mathbf{C}$,

(c) θ_{AC},

(d) $\mathbf{A} \times \mathbf{C}$,

(e) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$,

(f) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$,

(g) $\hat{\mathbf{x}} \times \mathbf{B}$, and

(h) $(\mathbf{A} \times \hat{\mathbf{y}}) \cdot \hat{\mathbf{z}}$.

3.4 Given vectors $\mathbf{A} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}} + \hat{\mathbf{z}}3$ and $\mathbf{B} = \hat{\mathbf{x}}3 - \hat{\mathbf{z}}2$, find a vector $\mathbf{C}$ whose magnitude is 6 and whose direction is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

3.5* Given $\mathbf{A} = \hat{\mathbf{x}}(2x + 3y) - \hat{\mathbf{y}}(2y + 3z) + \hat{\mathbf{z}}(3x - y)$, determine a unit vector parallel to $\mathbf{A}$ at point $P(1,-1,2)$.

3.6 By expansion in Cartesian coordinates, prove

(a) the relation for the scalar triple product given by Eq. (3.29), and

(b) the relation for the vector triple product given by Eq. (3.33).

*Answer(s) available in Appendix D.

Sections 3-2 and 3-3: Coordinate Systems

3.7* Convert the coordinates of the following points from Cartesian to cylindrical and spherical coordinates:

(a) $P_1(1,2,0)$,

(b) $P_2(0,0,3)$,

(c) $P_3(1,1,2)$, and

(d) $P_4(-3,3,-3)$.

3.8 Use the appropriate expression for the differential surface area ds to determine the area of each of the following surfaces:

(a) $r=3;\ 0 \le \phi \le \pi/3;\ -2 \le z \le 2$,

(b) $2 \le r \le 5;\ \pi/2 \le \phi \le \pi;\ z=0$,

(c) $2 \le r \le 5;\ \phi=\pi/4;\ -2 \le z \le 2$,

(d) $R=2;\ 0 \le \theta \le \pi/3;\ 0 \le \phi \le \pi$, and

(e) $0 \le R \le 5;\ \theta=\pi/3;\ 0 \le \phi \le 2\pi$.

Also sketch the outline of each surface.

3.9* Find the volumes described by

(a) $2 \le r \le 5;\ \pi/2 \le \phi \le \pi;\ 0 \le z \le 2$, and

(b) $0 \le R \le 5;\ 0 \le \theta \le \pi/3;\ 0 \le \phi \le 2\pi$.

Also sketch the outline of each volume.

3.10 Given vectors

$$\mathbf{A} = \hat{\mathbf{r}}(\cos\phi + 3z) - \hat{\boldsymbol{\phi}}(2r + 4\sin\phi) + \hat{\mathbf{z}}(r - 2z),$$
$$\mathbf{B} = -\hat{\mathbf{r}}\sin\phi + \hat{\mathbf{z}}\cos\phi,$$

find

(a) θ_{AB} at $(2,\pi/2,0)$, and

(b) a unit vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$ at $(2,\pi/3,1)$.

3.11* Determine the distance between the following pairs of points:

(a) $P_1(1,1,2)$ and $P_2(0,2,2)$,

(b) $P_3(2,\pi/3,1)$ and $P_4(4,\pi/2,0)$, and

(c) $P_5(3,\pi,\pi/2)$ and $P_6(4,\pi/2,\pi)$.

3.12 Transform the following vectors into cylindrical coordinates and then evaluate them at the indicated points:

(a) $\mathbf{A} = \hat{\mathbf{x}}(x+y)$ at $P_1(1,2,3)$,

(b) $\mathbf{B} = \hat{\mathbf{x}}(y-x) + \hat{\mathbf{y}}(x-y)$ at $P_2(1,0,2)$,

(c) $\mathbf{C} = \hat{\mathbf{x}}y^2/(x^2+y^2) - \hat{\mathbf{y}}x^2/(x^2+y^2) + \hat{\mathbf{z}}4$ at $P_3(1,-1,2)$, and

(d) $\mathbf{D} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta + \hat{\boldsymbol{\phi}}\cos^2\phi$ at $P_4(2,\pi/2,\pi/4)$.

3.13* Transform the following vectors into spherical coordinates and then evaluate them at the indicated points:

(a) $\mathbf{A} = \hat{\mathbf{x}}y^2 + \hat{\mathbf{y}}xz + \hat{\mathbf{z}}4$ at $P_1(1,-1,2)$,

(b) $\mathbf{B} = \hat{\mathbf{y}}(x^2+y^2+z^2) - \hat{\mathbf{z}}(x^2+y^2)$ at $P_2(-1,0,2)$, and

(c) $\mathbf{C} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi + \hat{\mathbf{z}}\cos\phi\sin\phi$ at $P_3(2,\pi/4,2)$.

Sections 3-4 to 3-7: Gradient, Divergence, and Curl Operators

3.14 Find the gradient of the following scalar functions:

(a) $T = 2/(x^2+z^2)$,

(b) $V = xy^2z^3$,

(c) $U = z\cos\phi/(1+r^2)$, and

(d) $W = e^{-R}\sin\theta$.

3.15 Follow a procedure similar to that leading to Eq. (3.82) to derive the expression given by Eq. (3.83) for ∇ in spherical coordinates.

3.16 For the scalar function $V = xy - z^2$, determine its directional derivative along the direction of vector $\mathbf{A} = (\hat{\mathbf{x}} - \hat{\mathbf{y}}z)$ and then evaluate it at $P(1,-1,2)$.

3.17* For the vector field $\mathbf{E} = \hat{\mathbf{x}}xz - \hat{\mathbf{y}}yz^2 - \hat{\mathbf{z}}xy$, verify the divergence theorem by computing

(a) the total outward flux flowing through the surface of a cube centered at the origin and with sides equal to 2 units each and parallel to the Cartesian axes, and

(b) the integral of $\nabla \cdot \mathbf{E}$ over the cube's volume.

3.18 For the vector field $\mathbf{E} = \hat{\mathbf{r}}10e^{-r} - \hat{\mathbf{z}}3z$, verify the divergence theorem for the cylindrical region enclosed by $r = 2$, $z = 0$, and $z = 4$.

3.19* A vector field $\mathbf{D} = \hat{\mathbf{r}}r^3$ exists in the region between two concentric cylindrical surfaces defined by $r = 1$ and $r = 2$, with both cylinders extending between $z = 0$ and $z = 5$. Verify the divergence theorem by evaluating

(a) $\oint_S \mathbf{D} \cdot d\mathbf{s}$, and

(b) $\int_{\mathcal{V}} \nabla \cdot \mathbf{D}\, d\mathcal{V}$.

3.20 For the vector field $\mathbf{D} = \hat{\mathbf{R}}3R^2$, evaluate both sides of the divergence theorem for the region enclosed between the spherical shells defined by $R = 1$ and $R = 2$.

3.21* For the vector field $\mathbf{E} = \hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)$, calculate

(a) $\oint_C \mathbf{E} \cdot d\mathbf{l}$ around the triangular contour shown in Fig. 3-25(a), and

(b) $\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s}$ over the area of the triangle.

3.22 Repeat Problem 3.21 for the contour shown in Fig. 3-25(b).

3.23* Verify Stokes's theorem for the vector field

$$\mathbf{B} = (\hat{\mathbf{r}}r\cos\phi + \hat{\boldsymbol{\phi}}\sin\phi)$$

by evaluating

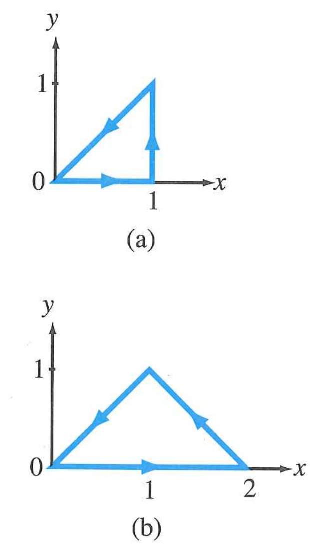

Figure 3-25: Contours for (a) Problem 3.21 and (b) Problem 3.22.

(a) $\oint_C \mathbf{B} \cdot d\mathbf{l}$ over the semicircular contour shown in Fig. 3-26(a), and

(b) $\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s}$ over the surface of the semicircle.

3.24 Repeat Problem 3.23 for the contour shown in Fig. 3-26(b).

3.25* Determine if each of the following vector fields is solenoidal, conservative, or both:

(a) $\mathbf{A} = \hat{\mathbf{x}}2xy - \hat{\mathbf{y}}y^2$,

(b) $\mathbf{B} = \hat{\mathbf{x}}x^2 - \hat{\mathbf{y}}y^2 + \hat{\mathbf{z}}2z$,

(c) $\mathbf{C} = \hat{\mathbf{r}}(\sin\phi)/r^2 + \hat{\boldsymbol{\phi}}(\cos\phi)/r^2$,

(d) $\mathbf{D} = \hat{\mathbf{R}}/R$.

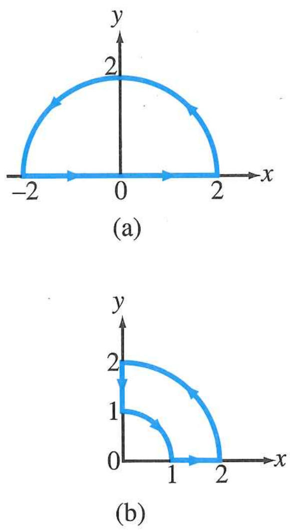

Figure 3-26: Contour paths for (a) Problem 3.23 and (b) Problem 3.24.

3.26 Find the Laplacian of the following scalar functions:

(a) $V = xy^2z^3$,

(b) $V = xy + yz + zx$,

(c) $V = 1/(x^2 + y^2)$,

(d) $V = 5e^{-r} \cos\phi$,

(e) $V = 10e^{-R} \sin\theta$.